Housing Rent, Inelastic Housing Supply, and International Business Cycles*

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Abstract

Despite their distinctive features—such as large expenditure shares and inelastic supply—housing services have received scant attention in the international macroeconomics literature. To fill this gap, I examine the role of housing in international business cycles for eurozone countries. I show that housing rents exhibit larger variations than the prices of tradables and other nontradables, both in cross-country and time-series. In addition, among all prices, housing rent stands out as the dominant contributor to both the Balassa-Samuelson effect and the negative Backus-Smith correlation. By simulating eurozone economies using a two-country model with a realistically calibrated housing sector, I show that the cross-country distribution of sectoral productivities, inelastic housing supply, and its interaction with the wealth effect via incomplete markets are crucial to understanding the empirical moments of real exchange rates. Compared with the standard model, the model with the housing sector generates larger real exchange rate variations, a more substantial Balassa-Samuelson effect, and more realistic Backus-Smith correlations.

Keywords: Real Exchange Rates, Balassa- Samuelson Effect, Backus-Smith Puzzle, Housing Market, Housing Rents, International Business Cycles

JEL Classification: F41, F44, R31

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1 Introduction

The real exchange rate (RER), defined as the relative price levels across countries, is a crucial general equilibrium object that influences core model mechanisms, including international risk sharing and trade. This indicates that a comprehensive grasp of the real exchange rate is essential for sound policy recommendations and legitimate academic research. Unfortunately, our understanding of real exchange rates is still limited, as evidenced by the various puzzles described by Itskhoki (2021).

One of the potential causes for the limited understanding of real exchange rates might stem from the abstraction of housing. Most international macroeconomic models neglect housing, assuming that housing services are the same as other nontradable services. However, housing deserves separate attention, as it is one of the most important components of household consumption. Households in European countries allocated approximately 20% of their disposable income to housing rents between 2014 and 2020, according to a EU Statistics of Income and Living Conditions Survey. Moreover, housing services differ markedly from other goods and services. Housing supply is highly inelastic due to long construction periods and heavy reliance on land as the primary input, coupled with limited land availability due to urbanization and stringent land-use regulations. Ignoring the economic significance and unique characteristics of housing services can lead to a limited understanding of international business cycles.

This paper addresses this gap by focusing specifically on housing services and distinguishing them from other nontradable services. In particular, I investigate the role of housing rent in three empirical aspects of real exchange rates: cross-sectional and time-series variations, the Balassa-Samuelson effect, and the Backus-Smith correlation. To conduct this analysis, I use the Eurostat-OECD Purchasing Power Parity database. This database covers 224 items and represents an entire consumption basket. Notably, it includes data on pure housing rent, excluding maintenance fees and utility costs. Furthermore, it not only tracks relative price changes but also provides relative price levels. By categorizing goods and services as tradable items, nontradable items, and housing services, I effectively decompose the aggregate real exchange rate into three components: tradable real exchange rate, nontradable real exchange rate, and rent real exchange rate. As a result, the aggregate real exchange rate becomes an expenditure-weighted sum of these three components; this creates an ideal environment for examining the role of rent in the dynamics of real exchange rates.

I focus on eurozone countries in which the nominal exchange rates among countries are set at one, which eliminates the influence of nominal exchange rates on real exchange rates. It is widely recognized that nominal exchange rates can be affected by monetary policies and financial shocks. If the real exchange rates were primarily driven by nominal exchange rates, this could complicate examination of the connection between the housing sector and real exchange rates. In addition, my goal is to investigate the real supply and demand aspects of housing services, rather than delving into housing-related financial market features such as mortgages. Consequently, the eurozone area is an ideal environment for this project. This strategy aligns with the approach used by Berka et al. (2018), which has proven fruitful.

Descriptive statistics analysis reveals that the rent real exchange rate exhibits larger variations and persistence in both cross-section and time-series than do tradables and nontradables. Furthermore, from the variance decomposition of the aggregate real exchange rate, it is shown that the rent real exchange rate contributes 33% of the aggregate real exchange rate variation across different countries (cross-sectional) and accounts for up to 60% of the aggregate real exchange rate variation over time (time-series), with the specific percentage varying by country. An intriguing observation is that in the time-series dimension, the rent real exchange rate displays very large fluctuations in countries significantly affected by demand shocks, such as Greece and Ireland, underscoring the importance of the inelastic nature of housing supply.

Furthermore, I augment this panel data on sectoral real exchange rates with data on relative real GDP per capita to investigate the Balassa-Samuelson effect. This effect is the empirical regularity with which countries with higher real GDP per capita tend to exhibit higher price levels, which is well documented by Rogoff (1996). The Balassa-Samuelson hypothesis is the most well-known theory to explain such a phenomenon. It posits that higher relative productivity in the tradable sector pushes up production factor prices, which in turn pushes up nontradable prices and the overall price level. Although this is considered to be applicable primarily between developed and developing countries in the long run, recent research by Berka et al. (2018) indicates that it also holds among eurozone countries in the short run. I extend their work by specifically examining the role of housing rent. My panel regression analysis reveals that a 1% higher real GDP per capita than the eurozone average corresponds to a 0.25% higher price level than the eurozone average. In addition, I further dissect the contribution of each sector's real exchange rate to this aggregate effect. Remarkably, even with the modest 16% expenditure weight associated with rent taken into account, 0.122% of the 0.25% total relative price increase can be attributed to the relative rent increase. This constitutes nearly half of the overall effect. This is because a 1% higher relative real GDP per capita translates to a 0.76% higher relative rent. These findings substantiate the significance of housing in the Balassa-Samuelson effect.

I also incorporate relative real consumption data in my dataset to examine the Backus-Smith correlation, which is the time-series link between the growth of the real exchange rate and real relative consumption. Typically, empirical data reveal this correlation to be close to zero or even negative. This suggests that countries' consumption increases more than foreign countries when their price levels increase more than the foreign countries, which implies a deviation from perfect risk sharing. Contrary to this, Backus and Smith (1993) demonstrated that a standard two-country model with a complete market predicts this correlation to be 1. This became a significant puzzle when attempts to modify the model's prediction closer to data proved difficult, even under the assumption of an incomplete market. Several promising solutions have been proposed—yet no previous research has considered the impact of the rent real exchange rate. In fact, the rent real exchange rate turns out to be pivotal. My panel regression analysis indicates that when a country's consumption grows 1% more than the eurozone average, its real exchange rate appreciates by 0.14%, which implies a negative Backus-Smith correlation. Strikingly, 0.126% of this 0.14% appreciation stems from the rent real exchange rate, even taking into account the relatively low expenditure weight of housing rent.

Motivated by this empirical evidence, I develop a two-country model that incorporates a realistically calibrated housing sector by combining two models, Berka et al. (2018) and Davis and Heathcote (2005). Also, I assume an incomplete market between countries to examine the role of the wealth effect studied by Corsetti et al. (2008), since an inelastic supply of housing naturally implies the importance of a demand shock via the wealth effect. As Itskhoki (2021) underscores, real exchange rates are shaped through general equilibrium forces. This requires examination of real exchange rates from a general equilibrium viewpoint. To achieve this, I simulate my model using sectoral productivity shocks—namely, those in the tradable, non-tradable, and construction sectors—directly calibrated from the EUKLEMS database. By simulating under varied calibrations of the housing market, I delve into the role housing plays in the dynamics of real exchange rates.

Simulations of my model reveal its capability to generate significant variations in the real exchange rates. Notably, the inelastic supply of housing services—attributed to land being a primary input for construction and the small flow of new housing relative to existing housing stock—reduces the variations of the real exchange rate that arises from the productivity shocks under a complete market. However, it amplifies the impact of relative demand changes from the wealth effect of Corsetti et al. (2008) under an incomplete market due to its inelastic supply.

Model simulations also provide valuable insights on the Balassa-Samuelson effect. Simulations show that housing's heavy reliance on land as the primary input actually dampens the textbook Balassa-Samuleson hypothesis mechanism because residential-zoned land is not used in the tradable sector in the model. However, the model still generates the strong Balassa-Samuelson effect via housing rents as in the data, and it comes from the cross-country distribution of sectoral productivities. The sectoral productivity levels of eurozone countries, directly calibrated from the EUKLEMS database, reveal a pattern in which countries with highly productive tradable sectors also tend to have highly productive nontradable sectors, but a relatively less productive construction sector, allowing the textbook Balassa Samuelson hypothesis mechanism to work mainly through the rent real exchange rate. This observation aligns with the recent findings of stagnant productivity in the construction sector documented by Goolsbee and Syverson (2023).

Lastly, incorporating a housing sector improves my model's ability to address the Backus-Smith puzzle. Unlike the standard model, mine accurately replicates the panel regression results in the data and generates most of the negative Backus-Smith correlation through the rent real exchange rate component. This is primarily because inelastic housing supply renders the aggregate supply more inelastic, in turn causing the aggregate price level to be more responsive to relative demand shifts. Consequently, price levels rise more when relative consumption increases via demand shifts from the wealth effect, which generates a stronger negative Backus-Smith correlation. Furthermore, the now more inelastic aggregate supply diminishes the impact of nontradable sector and construction sector productivity shocks that typically act as potent supply shocks and generate a positive Backus-Smith correlation. Last but not least, through the substitution effect, housing in the model helps the model match not only the aggregate but also other sectoral Backus-Smith correlations.

This paper builds on a large literature on the secular movements of the real exchange rate. Engel (1999) documented that the bulk of US real exchange rate variation comes from relative prices of the tradable sector, under a floating exchange rate regime. Several studies suggest that differences in the consumer prices of traded goods across countries are due to the distribution margin (e.g., Burstein et al. 2005, Betts and Kehoe 2006). On the other hand, other studies analyze firms' pricing behaviors based on variable markups (e.g., Atkeson and Burstein 2008). Beyond these, Mussa (1986) documented large real exchange rate volatility under a floating exchange rate compared with that under a fixed exchange rate regime. My paper contributes to the literature by examining the importance of housing rent in real exchange rate movements under a fixed exchange rate regime.

My paper also intersects with the extensive literature on the Balassa-Samuelson effect. Rogoff (1996) validated that countries with higher real GDP per capita, which is employed as a proxy for tradable sector productivity, exhibit more appreciated real exchange rates in a cross-sectional analysis of 1990 data. Bordo et al. (2017) identified a long-run correlation between relative income and real exchange rates across a panel of 14 countries in relation to the US. Several other studies have directly probed the nexus between real exchange rates and sectoral productivities, resulting in a spectrum of outcomes (e.g., Choudhri and Schembri 2014, Gubler and Sax 2019). A recent contribution, Berka et al. (2018), examines eurozone countries' real exchange rates and sectoral productivities. Their findings suggest that the Balassa-Samuelson effect permeates the eurozone—even in the short run and within a time-series framework—when factoring in the labor wedge. I extend their work by focusing on housing rent and show that housing contributes to over half of the entire Balassa-Samuelson effect. By simulating the model with a realistically calibrated housing sector, I show that the cross-country distribution of sectoral productivities and the wealth effect are the key forces for the Balassa-Samuelson effect in eurozone countries.

Lastly, this paper builds on literature on the Backus-Smith puzzle. While the correlation between real exchange rates and relative consumption is negative or near zero in the data, Backus and Smith (1993) found that a standard two-country model with a complete market predicts this correlation will be 1. Because this model prediction largely depends on the complete market assumption, Chari et al. (2002) constructed a two-country model with an incomplete market under monetary policy shocks. However, they again generated a correlation closer to 1. Later, Corsetti et al. (2008) generated a negative correlation under an incomplete market by assuming either very persistent productivity shocks or very low substitutabiltiy between home and foreign tradable goods. Benigno and Thoenissen (2008) generated a negative correlation by using the Balassa-Samuelson mechanism. Other papers use home production (e.g., Karabarbounis 2014) or non-rational expectations (e.g., Lambrias 2020) to resolve the Backus-Smith puzzle. Devereux and Hnatkovska (2020) point out that the nominal exchange rate is important for a negative Backus-Smith correlation. The most recent contribution to the literature is Itskhoki and Mukhin (2021), who resolve many international macroeconomic puzzles via financial frictions. My paper contributes to this literature by analyzing the role of housing rents in the Backus-Smith puzzle. While the importance of nominal exchange rate has been discussed in the literature, my paper shows that there are still negative Backus-Smith correlations among eurozone countries. Moreover, I show that, among all relative prices, the rent real exchange rate contributes most. Lastly, I theoretically contribute to this literature by showing how a realistically calibrated housing sector can help the standard model produce improved predictions for the Backus-Smith puzzle.

The paper is structured as follows. Section 2 details the data sources and describes how I construct sectoral real exchange rates. It also states the basic properties of sectoral real exchange rates and includes panel regression analyses that identify the role of the rent real exchange rate in both the Balassa-Samuelson effect and the Backus-Smith puzzle. Section 3 outlines the model that incorporates a housing sector. Section 4 presents the calibration of the model and shock processes. It also reports the simulation analysis result to elucidate the role of the housing sector in real exchange rate dynamics. Finally, Section 5 concludes and proposes future research questions.

2 Data: Housing Rents and Real Exchange Rates

2.1 Real Exchange Rates in Eurozone Countries

Data Source and Coverage I construct the aggregate and sectoral real exchange rates of eurozone countries using the Eurostat-OECD Purchasing Power Parity (Eurostat PPP) database, which contains the cross-country relative price levels of 224 items and covers a whole consumption basket of European countries. These include all types of goods and services, such as food, clothing, transportation, education, and health care services. Most importantly, they provide the relative prices of *Actual Rentals for Housing* and *Imputed Rentals for Housing*. These two relative prices are for housing rents that do not include any other costs, such as maintenance fees or utility costs. This enables cleaner identification of housing service prices. The full list of goods and services in the database is in the online appendix. Reporting frequency is annual. I use data from 2000 to 2019 to examine the period after the introduction of the euro and before the COVID-19 outbreak. In addition, I use data only on the 12 countries that introduced the euro in 2000: Austria, Belgium, Germany, Greece, Spain, Finland, France, Ireland, Italy, Luxembourg, the Netherlands, and Portugal.

It is worth emphasizing the quality of the data. As is well explained by Berka et al. (2018), the Eurostat PPP database offers a number of advantages compared with the datasets used in other research. First, to construct the Eurostat PPP database, each country conducts a national survey that covers all items in their consumption baskets. This implies that the database covers the overall price levels of the economy. Compared with research that uses price data from a single supermarket chain (Gopinath et al. 2011), from a single international retailer of household goods (Baxter and Landry 2017), or from a few online retailers (Cavallo et al. 2014), the Eurostat PPP database offers better coverage. Second, the Eurostat PPP database guarantees better cross-country comparability. For example, though some research has used price data that cover a comprehensive set of items, such as the Economist Intelligence Unit survey (Engel and Rogers 2004, Crucini and Shintani 2008), such data lack the validity of cross-country comparability. In contrast, the Eurostat PPP database is organized under a single entity, Eurostat, which guarantees more homogeneous data collection procedures across countries (e.g., the selection of items and outlets where prices are measured). In addition, the Eurostat PPP database undergoes an internal review process every year to check the comparability and completeness of the dataset. The fact that I use only eurozone countries also increases cross-country comparability, since they share similar cultural and legal backgrounds. In particular, housing rent data in the Eurostat PPP database offers the most credible cross-country comparability. The housing rent level is notoriously difficult to measure for cross-country comparison.

To overcome this, every year Eurostat asks all member countries to derive rent-level data based on their internal rent survey. Most reporting countries use rent survey data for their national account construction, demonstrating how precise and detailed the data are. In addition, Eurostat provides members with details on how to compute the rent price level.[1]

Construction of Real Exchange Rates This database provides the price level index (PLI) for 224 items that cover a whole consumption basket. A PLI (p_{ijt}) for item i and country j is defined as the log relative price of item i in country j relative to that of the EU 15 average (geometric average).

$$p_{ijt} = log(\frac{P_{iEU15t}}{P_{ijt}}) \quad \text{where} \quad P_{iEU15t} = \prod_{k \in EU} P_{ikt}^{\frac{1}{15}}.$$

For example, if the croissant price is 1.2 euros in France but its EU 15 average price is 1 euro, the PLI of croissants in France is given as $0.079 = log(\frac{1.2}{1.0})$. Note that this contains information on the relative levels of the prices. For every item in the consumption basket, I can observe how expensive that item is in a certain country compared with the EU 15 average.

The database also contains the expenditure weight of each item for each country. As is usual, the expenditure weight (γ_{ijt}) for item i and country j is defined as follows:

$$\gamma_{ijt} = \frac{EXP_{ijt}}{\sum_{i=1}^{224} EXP_{ijt}}, \quad \sum_{i=1}^{224} \gamma_{ijt} = 1.$$

By using these PLIs and expenditure weights, I construct aggregate real exchange rates as follows. Note that since PLI is defined as the price level compared with the EU 15 average, this real exchange rate is between country j and the EU 15 average:[2]

$$q_{j,t} = \sum_{i=1}^{224} \gamma_{ijt} p_{ijt} = log(\frac{\prod_{i=1}^{224} P_{iEU15t}^{\gamma_{ijt}}}{\prod_{i=1}^{224} P_{ijt}^{\gamma_{ijt}}}).$$

In this definition, the real exchange rate is effectively the expenditure-weighted geometric average of the relative prices of goods and services. Intuitively, this implies the relative overall price level of country j compared with that of the EU 15 average. Going one step further, I can decompose this aggregate real exchange rate into

[1] While collecting rent data, the quality of houses is also taken into account. They are classified by the number of rooms, type of house (apartment, single house, etc.), and features (central heating system, etc.) to derive a quality-based quantity index. For more information, refer to the OECD and Eurostat (2012)

[2] This means that I use the country j's expenditure weights to calculate the average price level of the EU 15 countries. As will be shown in Figure 1, there is not much cross-country difference in expenditure weights.

sectoral real exchange rates as follows:

$$q_{jt}^{T} = \frac{\sum_{i \in T} \gamma_{ijt} p_{ijt}}{\sum_{i \in T} \gamma_{ijt}} \quad \text{(159 Items)}, \quad (\sum_{i \in T} \gamma_{ijt} = \gamma_j^T),$$
$$q_{jt}^{NT} = \frac{\sum_{i \in NT} \gamma_{ijt} p_{ijt}}{\sum_{i \in NT} \gamma_{ijt}} \quad \text{(63 Items)}, \quad (\sum_{i \in NT} \gamma_{ijt} = \gamma_j^{NT}),$$
$$q_{jt}^{R} = \frac{\sum_{i \in H} \gamma_{ijt} p_{ijt}}{\sum_{i \in H} \gamma_{ijt}} \quad \text{(2 Items)}, \quad (\sum_{i \in H} \gamma_{ijt} = \gamma_j^R = 1 - \gamma_j^T - \gamma_j^{NT}).$$

In essence, these are again the expenditure-weighted geometric averages of the prices of a certain group of goods and services. It shows that I classify 159 items as tradable, 63 items as nontradable, and 2 items as housing rents. For this classification, I closely follow the approach of Berka et al. (2018).[3] Two housing-service related items are *Actual Rentals for Housing* and *Imputed Rentals for Housing*, meaning that I use both owners' housing costs and renters' housing costs. By construction, I arrive at the following decomposition of the aggregate real exchange rate:

$$q_{jt} = \gamma_j^T q_{jt}^T + \gamma_j^{NT} q_{jt}^{NT} + \gamma_j^R q_{jt}^R,$$

Note $q_{jt} < 0$ implies that country j's overall price level is higher than the EU 15 average, and $\Delta q_{jt} < 0$ implies the appreciation of country j's real exchange rate.

Properties of Real Exchange Rates and Expenditure Weights I start with the descriptive statistics in Table 1. The upper panel of the table provides information on each country and the lower panel provides information on the averages of all countries. Together, they provide the overview on the real exchange rates in the eurozone.

The *Mean* table in the upper panel shows how each country's sectoral price level compares with the EU 15 average. Ireland is in the first row, with a 13.2% higher price level, and Portugal is in the last row with a 24.4% lower price level than the EU 15 average. An interesting observation is that countries with a higher overall price level have higher prices for nontradables and housing rent.

To understand the overall characteristics of sectoral real exchange rates, see the lower panel of Table 1. The first important observation is the q^R shows the largest volatility in both the cross-section and time-series compared with q^T and q^{NT}. As is well known, housing service supply is inelastic, and thus any demand shocks likely generate a large price response rather than a quantity response. Lastly, q^R is also the most persistent compared with other sectoral productivities. Because of the slow response of supply, price changes via shocks are likely to last longer.

Four subplots on the left side of Figure 1 shows movements of the sectoral real exchange rates. I set the ranges of the y-axes in all graphs the same to facilitate comparison across sectors. An interesting pattern is that the variation of q^T is the smallest

[3] The classification procedure is detailed in the online appendix.

Table 1: Descriptive Statistics of Real Exchange Rates

	Mean				*Standard deviation*				*Autocorrelation*(1)			
Country	$\bar{q}$	$\bar{q^T}$	$\bar{q^{NT}}$	$\bar{q^R}$	$std(q)$	$std(q^T)$	$std(q^{NT})$	$std(q^R)$	$\rho(q)$	$\rho(q^T)$	$\rho(q^{NT})$	$\rho(q^R)$
Ireland	-0.132	-0.102	-0.140	-0.187	0.034	0.021	0.035	0.128	0.737	0.500	0.731	0.866
Finland	-0.124	-0.093	-0.138	-0.187	0.021	0.028	0.034	0.022	0.823	0.919	0.725	0.681
Luxembourg	-0.047	0.080	-0.059	-0.425	0.040	0.015	0.087	0.039	0.954	0.692	0.965	0.564
France	0.002	0.023	0.002	-0.057	0.014	0.027	0.034	0.030	0.536	0.813	0.801	0.888
Belgium	0.005	0.006	0.006	-0.003	0.012	0.019	0.017	0.028	0.677	0.736	0.774	0.899
Netherlands	0.010	0.027	0.010	-0.038	0.026	0.015	0.035	0.055	0.866	0.585	0.770	0.954
Austria	0.028	0.017	-0.047	0.273	0.015	0.014	0.018	0.053	0.715	0.690	0.732	0.920
Germany	0.030	0.033	0.029	0.020	0.023	0.015	0.028	0.068	0.912	0.644	0.885	0.979
Italy	0.068	0.008	0.100	0.222	0.018	0.018	0.021	0.049	0.693	0.723	0.416	0.682
Spain	0.162	0.147	0.176	0.172	0.032	0.025	0.047	0.070	0.858	0.877	0.814	0.869
Greece	0.211	0.134	0.254	0.364	0.050	0.041	0.062	0.200	0.863	0.916	0.839	0.944
Portugal	0.244	0.118	0.313	0.641	0.016	0.022	0.045	0.121	0.530	0.607	0.768	0.965

Aggregate	$std(mean_i)$	$mean(std_i)$	$mean(autocorr_i)$
q	0.119	0.025	0.764
q^T	0.079	0.022	0.725
q^{NT}	0.144	0.039	0.768
q^R	0.286	0.072	0.851

$q_{it} = ln(P_{EU15t}/P_{it})$, $q^T_{it} = ln(P^T_{EU15t}/P^T_{it})$, $q^{NT}_{it} = ln(P^{NT}_{EU15t}/P^{NT}_{it})$, $q^R_{it} = ln(P^R_{EU15t}/P^R_{it})$, where P_{EU15t} is the aggregate price level of 15 European countries. The data period is from 2000 to 2019 and data are at annual frequency. Countries in the upper panel are sorted in the order of price levels.

in both the cross-section and time-series.. In addition, it even shows the sign of the convergence of price levels as the year progresses. Most importantly, the large variation of q^R in both the cross-section and time-series exist, as in Table 1. These volatile dynamics of q^R are most prominent in countries that experienced significant demand shocks during the eurozone crisis (e.g., Portugal, Ireland, and Greece).

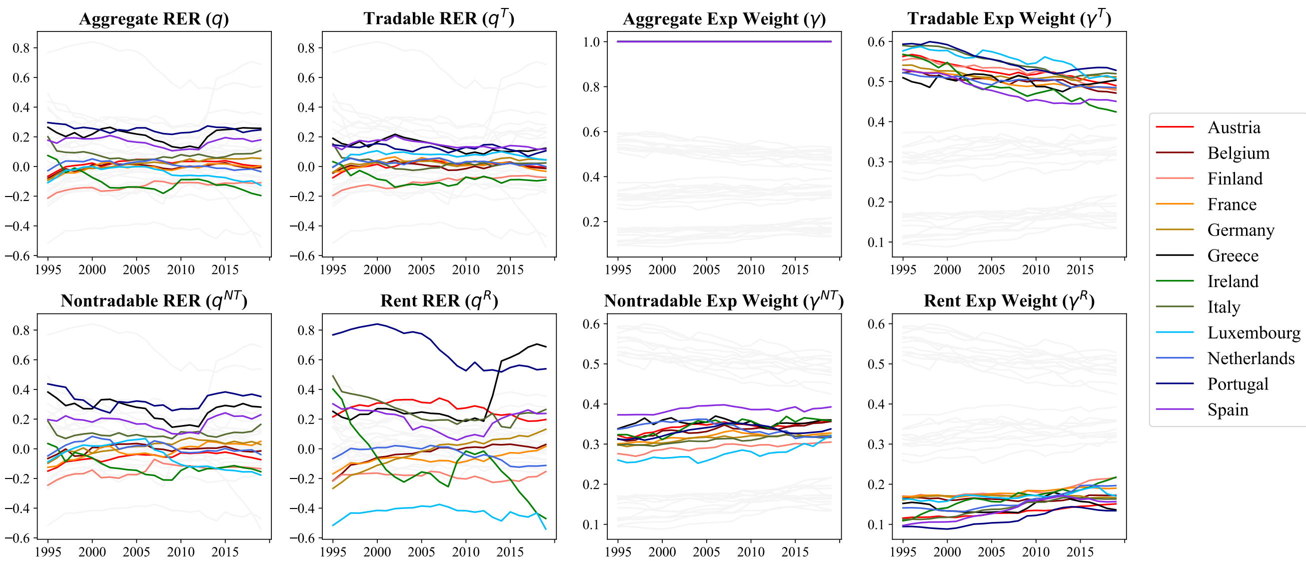

Figure 1: Sectoral Real Exchange Rates and Expenditure Weights

The other four subplots on the right side of Figure 1 show the dynamics of the sectoral expenditure weights of all countries. Again, I set the ranges of the y-axes in all graphs the same to facilitate comparison across sectors (except for the total expenditure weight, which is 1 by definition.) A notable pattern is that the expenditure weights of tradables are decreasing, while the expenditure weights for nontradables and housing rents are increasing. Also, note that rent expenditure weights are roughly in the range of 15%-20%, which is a substantial weight for only two items. Though there is cross-country heterogeneity in expenditure weights, overall heterogeneity is not that significant. In addition, the relative size of each sector's expenditure weight is roughly the same across countries.

Variance Decomposition While the q^R shows very large cross-country and time-series variations, it also has the lowest expenditure weight compared with tradables and nontradables. Thus, it might be the case that in the end, relative rents do not affect the dynamics of the aggregate real exchange rate much. To get a sense of the quantitative importance of the relative rents, I conduct the following variance decomposition. Any variance of $Var(q)$ can be decomposed as follows:

$$Var(q) = Cov(q, \gamma^T q^T + \gamma^{NT} q^{NT} + \gamma^R q^R) = \gamma^T Cov(q, q^T) + \gamma^{NT} Cov(q, q^{NT}) + \gamma^R Cov(q, q^R).$$

By dividing both sides by $Var(q)$, I arrive at the following:

$$1 = \gamma^T Corr(q, q^T) \frac{std(q^T)}{std(q)} + \gamma^{NT} Corr(q, q^{NT}) \frac{std(q^{NT})}{std(q)} + \underbrace{\gamma^R Corr(q, q^R) \frac{std(q^R)}{std(q)}}_{\text{Share of } q^R \text{ in } RER \text{ Variation}}.$$

According to this decomposition, the contribution of the rent real exchange rate can be calculated as $\gamma^R Corr(q, q^R) \frac{std(q^R)}{std(q)}$. Note that this measure also takes the expenditure weight into account. I apply this decomposition to both cross-section and time-series variation. For the cross-section, I decompose the cross-country variation of the average aggregate real exchange rate, $Var(\bar{q})$. This exercise demonstrates how important housing rent is in accounting for price-level differences across countries. The left panel in Figure 2 shows the result. The rent real exchange rate accounts for 33% of the total variations. Considering its 16% expenditure weight, which is substantially smaller than that of tradables and non-tradables, the fact that q^R accounts for one-third of the total variation is remarkable.

For time-series variation, such decomposition is applied to each country j's time-series variation, $Var(q_{jt})$. The right panel in Figure 2 shows the results. There are countries in which more than half of the total variation comes from rent real exchange rates. Again, given the 15%-20% expenditure weight of housing rents, this shows the significant role of rent real exchange rates.

This is in contrast to the findings of Engel (1999), whereby the relative price of the tradable is the one that drives the time-series standard deviation of the real exchange rate. This is probably because Engel (1999) examined the floating exchange rate regime. Berka et al. (2018) examined the important role of the relative price of

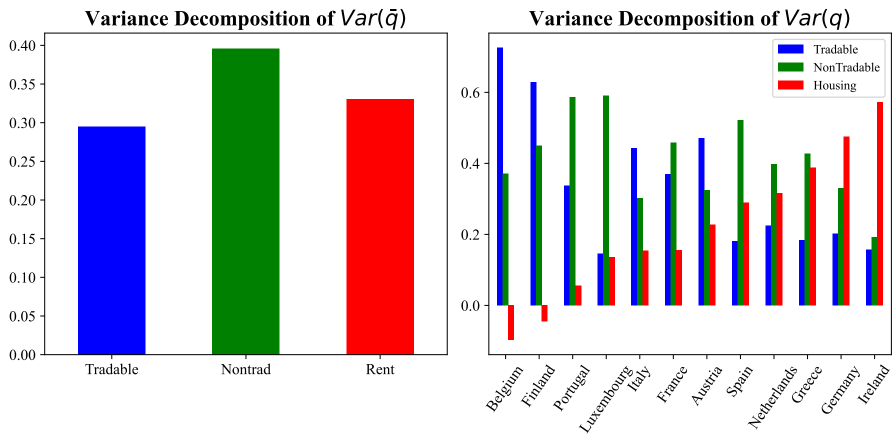

Figure 2: Variance Decomposition of RER

the nontradable under fixed exchange rates. However, they did not decompose it into housing and non-housing components, thereby overlooking the importance of housing rents. Building on their work, I further dissect the nontradable prices into non-housing nontradable prices and housing rents. I demonstrate that the significance of housing rents, which are arguably influenced by different mechanisms compared to other nontradables, is comparable to that of all other non-housing nontradables. This result implies that housing rent deserves more attention in the real exchange rate literature. In addition, the rent expenditure weights provided by the Eurostat PPP database are much lower than those from the household survey.[4] This implies that any empirical implications of q^R I find with the Eurostat PPP database can be considered to be a lower bound.

2.2 The Balassa-Samuelson Effect and the Backus-Smith Puzzle

In this subsection, we examine the role of housing rents in the Balassa-Samuelson effect and the Backus-Smith correlation by combining my panel dataset of sectoral real exchange rates with those of real GDP per capita and real consumption. Both datasets are from the national account of each country and are in real terms to measure volume changes.[5] To make these variables consistent with real exchange rates, I

[4] For European countries, households' actual expenditure weights on rents are much larger than Eurostat PPP data imply if I use EU-SILC household survey data. The rent expenditure weight is higher for poorer households.

[5] For real GDP per capita, I use *real GDP per capita in PPP-adjusted EU15* and, for real relative consumption, I use *real final consumption expenditure of households, chain-linked volumes (2010), million euro.* Some prior research has used per capita consumption for the Backus-Smith correlation, and the results do not change when I use per capita consumption. Tables based on per capita consumption are provided in the online appendix.

define relative real GDP per capita (y) and relative real consumption (c) as follows:

$$y_{jt} = log(Y_{jt}/Y_{EU12t}),\ c_{jt} = log(C_{jt}/C_{EU12t}).$$

Note that Y_{EU12t} and C_{EU12t} are the geometric averages of 12 eurozone countries' real GDP per capita and real aggregate consumption. Δy_{jt} (Δc_{jt}) represents the relative growth rate of real GDP per capita (real aggregate consumption) relative to that of the EU 12 average. Unlike the case of real exchange rates, $\Delta y_{jt} > 0$ ($\Delta c_{jt} > 0$) implies that country j's real GDP per capita (real aggregate consumption) is growing faster than the average of the 12 eurozone countries.

Cross-sectional Variation and Time-series Variation To examine the Balassa-Samuelson effect, Rogoff (1996) used the cross-country data in 1990 while Corsetti et al. (2008) calculated each country's time-series correlation between its real exchange rate and relative consumption growth, effectively exploiting the time-series dimension. While these two moments concern variations in different dimensions, what I have is panel data that capture both cross-section and time-series variations. Running a pooled OLS might prevent me from observing the empirical patterns of interest.

To avoid such problems, I run the following four regressions:

$$\begin{aligned}
\bar{q}_j &= \alpha + \beta \bar{x}_t + \eta_t && \text{(Country Average)}, && (1)\\
q_{jt} &= \alpha_t + \beta x_{jt} + \epsilon_{jt} && \text{(Time Fixed Effect)}, && (2)\\
\Delta q_{jt} &= \alpha + \Delta \beta x_{jt} + \epsilon_{jt} && \text{(Growth Rate)}, && (3)\\
q_{jt} &= \alpha_j + \beta x_{jt} + \epsilon_{jt} && \text{(Country Fixed Effect)}. && (4)
\end{aligned}$$

The regressions in Equation (1) and (2) capture cross-sectional level variations. By averaging out across time in each country or by using the time-fixed effect, I remove time-series variations and capture only cross-country variations. The Balassa-Samuelson effect is expected to appear in these two regressions, and the Backus-Smith correlation is expected not to appear. By contrast, the regressions in Equation (3) and (4) capture time-series variations. By using its growth rate or by using the country-fixed effect, I remove the cross-country level variations.[6] Negative Backus-Smith correlations are expected to appear in these regressions.

Note that fixed effects are not used to remove any potential endogeneity stemming from unobserved heterogeneity. These are primarily to study the correlations, and these fixed effects are used to capture the variations of interest in different dimensions. I apply all of these regressions to both the Balassa-Samuelson effect and the Backus-Smith correlations, and see from which variations those relationships emerge.

[6]There is no significant cross-country heterogeneity in the mean growth rates of aggregate and sectoral real exchange rates.

Regression-based Decomposition Before directly jumping into the actual regression, I explain how I capture the role of rent real exchange rate. In fact, regression analysis provides very intuitive decomposition of the relationships. For example, if I am interested in the relationship between a variable of interest (x) and the aggregate real exchange rate (q), I can perform the following regression analysis:[7]

$$q_{jt} = \alpha + \beta x_{jt} + \epsilon_{jt}.$$

In this regression, β, which is $\frac{Cov(q,x)}{Var(x)}$, summarizes the relationship between q and x. Also, to examine the relationship between x and each sector's real exchange rate (q^T, q^{NT}, q^R), I can perform the following regression analysis:

$$q^k_{jt} = \alpha + \beta^k x_{jt} + \epsilon_{jt} \text{ for } k \in \{T, NT, R\}$$

Then, given that $q = \gamma^T q^T + \gamma^{NT} q^{NT} + \gamma^R q^R$ and the linearity of the OLS estimator, $\beta = \gamma^T \beta^T + \gamma^{NT} \beta^{NT} + \gamma^R \beta^R$. This procedure effectively decomposes the relationship between x and q represented by β into the weighted sum of the relationship between x and each sector's real exchange rate. This regression-based decomposition yields intuitive assessment of the role of each sector's real exchange rate. $\gamma^R \beta^R$ will be the contribution of q^R to the aggregate empirical relationship summarized by β. By using this decomposition, I will examine how much the rent real exchange rate contributes to the Balassa-Samuelson effect and the Backus-Smith correlation.

Housing Rents and the Balassa-Samuelson Effect First, I examine the Balassa-Samuelson effect in eurozone countries. As a motivating figure, I generate a scatter plot in which the average relative real GDP per capita of each country $(\bar{y}_j)$ is on the x-axis and each country's average aggregate real exchange rate $(\bar{q}_j)$ and sectoral real exchange rates $(\bar{q}^T_j, \bar{q}^{NT}_j, \bar{q}^R_j)$ are on the y-axis in Figure 3.

As in the left panel, countries with higher relative GDP per capita show higher relative price levels ($q < 0$). This implies that the Balassa-Samuelson effect exists in eurozone countries. Then, the right panel shows where such a relationship comes from. The relative price levels of tradables (q^T, denoted by the blue line) do not increase much, even if the country has a high GDP per capita. On the other hand, rent real exchange rates (q^R, denoted in yellow) exhibit a steep increase and decrease depending on the country's relative GDP per capita, implying the important role of relative rents in the Balassa-Samuelson effect.

To examine the data more systematically, by using four types of regressions stated in equation (1), (2), (3), and (4), I examine the relationship between aggregate and sec-

[7] This applies to all four regression forms discussed in the previous paragraph because all regressions are effectively OLS regressions in levels, growth rates, or demeaned values.

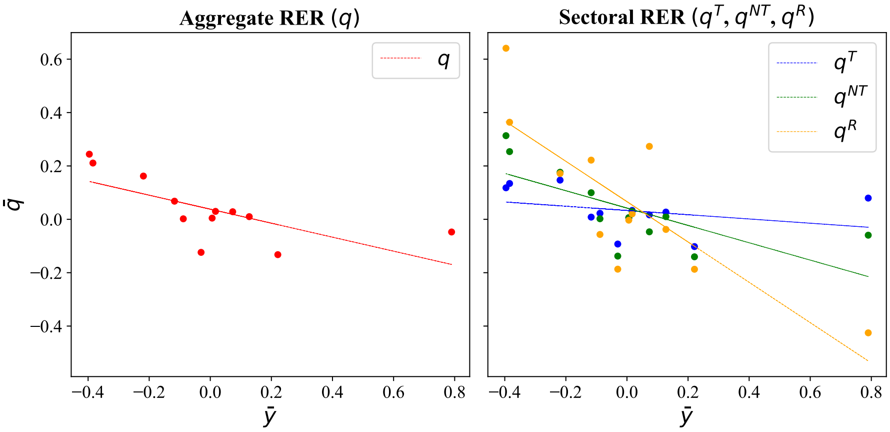

Figure 3: The Balassa-Samuelson Effect in Eurozone Countries

toral real exchange rates (q, q^T, q^{NT}, q^R) and relative real GDP per capita (y). Results of the regressions are reported in Table 2.

Table 2: Balassa-Samuelson Effect Regressions

		Cross-section						Time-series			
		$\bar{q}$	$\bar{q}^T$	$\bar{q}^{NT}$	$\bar{q}^R$			Δq	Δq^T	Δq^{NT}	Δq^R
Country Average	$\bar{y}$	-0.26*	-0.08	-0.33*	-0.76***	Growth Rate	Δy	0.07*	0.11***	0.13**	-0.17**
		(0.14)	(0.13)	(0.18)	(0.19)			(0.04)	(0.03)	(0.07)	(0.08)
	R^2	0.43	0.08	0.45	0.64		R^2	0.02	0.04	0.02	0.02
	N	12	12	12	12		N	240	240	240	240
		q	q^T	q^{NT}	q^R			q	q^T	q^{NT}	q^R
Time Fixed Effect	y	-0.26***	-0.07***	-0.31***	-0.75***	Country Fixed Effect	y	-0.11***	0.08*	-0.08	-0.67***
		(0.01)	(0.01)	(0.01)	(0.03)			(0.04)	(0.05)	(0.10)	(0.22)
	R^2	0.45	0.08	0.45	0.64		R^2	0.07	0.07	0.02	0.25
	N	240	240	240	240		N	240	240	240	240

$q = ln(P_{EU15t}/P_{it})$, $q^T = ln(P^T_{EU15t}/P^T_{it})$, $q^{NT} = ln(P^{NT}_{EU15t}/P^{NT}_{it})$, $q^R = ln(P^R_{EU15t}/P^R_{it})$ where P_{EU15t} is the price level of 15 European countries' average. $c = ln(C_{it}/C_{EU12t}), y = ln(Y_{it}/Y_{EU12t})$ where C_{EU12t}, Y_{EU12t} are geometric averages of C, Y over 12 eurozone countries. Data period is from 2000 to 2019 and data are at annual frequency. All standard errors are computed using a panel-corrected standard errors method under the assumption of period correlation (cross-sectional clustering). Parentheses below estimates include standard deviations. * means 10% significance, ** means 5% significance, *** means 1% significance.

In the first two regressions that capture cross-sectional variations, as expected, β is estimated as -0.26 statistically significantly. This implies that a country with 1% higher relative GDP per capita has the 0.26% higher relative price levels, proving the existence of the Balassa-Samuelson effect in eurozone countries. Another interesting observation is that q^R has very large coefficients compared with the other sectoral real exchange rates and β^T is not even significant in the country-average regression.

The two remaining regressions, which capture time-series variations, offer further evidence of the significance of housing. Given that the Balassa-Samuelson effect is widely accepted as a long-term empirical pattern (Rogoff 1996), unsurprisingly that the overall Balassa-Samuelson effects observed in these regressions are not pronounced.[8] Surprisingly, housing continues to play a significant role even in short-

[8] Berka et al. (2018) show that the Balassa-Samuelson hypothesis actually also works in dynamics. The difference between their regressions and mine is that they directly use sectoral productivity levels

term, year-to-year fluctuations, with countries experiencing rapid growth also seeing faster increases in housing rents.

Given that the cross-sectional pattern is stronger, I conduct the regression-based decomposition described in the previous paragraph for two regressions for cross-sectional variations. The left panel in Figure 4 shows the result. Each bar is each sectoral real exchange rate's β multiplied by the sectoral expenditure weight γ, so the sum of the blue, green, and red bar should be equal to the black bar.

Because β^T is not significant, this decomposition shows that almost half of the Balassa-Samuelson effect actually comes from the rent real exchange rate. While the rent expenditure weight is less than half the expenditure weight of other nontradables, its contribution to the Balassa-Samuelson effect is more than that. This implies the important role of housing rent in the Balassa-Samuelson effect.

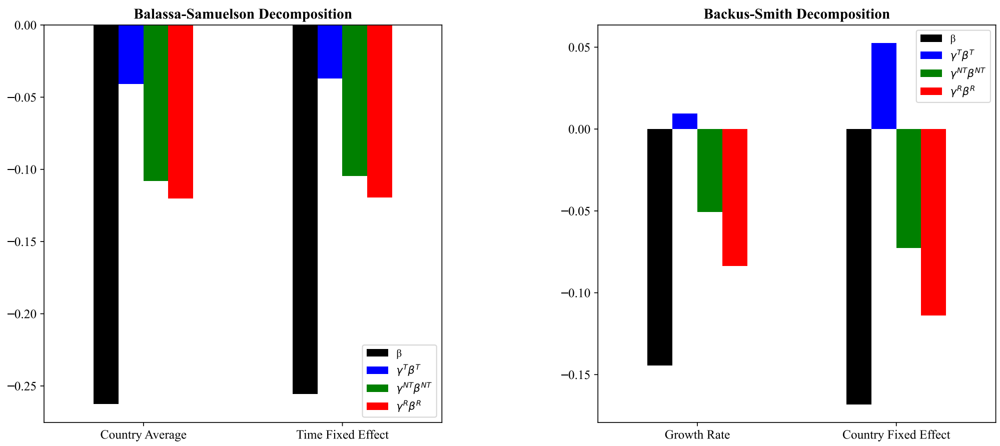

Figure 4: β Decompositions: Balassa-Samuelson Effect and Backus-Smith Puzzle

Housing Rents and Backus-Smith Correlations For the last empirical analysis, I examine the role of housing rents in the Backus-Smith correlation. Before moving to the regression analysis, following the literature, I calculate the Backus-Smith correlation of the aggregate and sectoral real exchange rates for each country in the eurozone. Table 3 shows these correlations for each country. Nine countries out of 12 show the near-zero or negative correlations between real relative consumption and real exchange rates, manifesting the presence of negative Backus-Smith correlations among eurozone countries. On average, the Backus-Smith correlations are -0.059, far from the 1 implied by the standard model.[9] In addition, we can see that Δq^R has a

from the EUKLEMS in the data and not relative real GDP per capita.

[9] Devereux and Hnatkovska (2020) argue that following the introduction of the euro, the average Backus-Smith correlations across eurozone countries turned positive, increasing from -0.19 to 0.05, compared to the pre-euro period. They also found that eight out of the twelve eurozone countries exhibited positive Backus-Smith correlations after the introduction of the euro by using data from 2000 to 2007. However, in my dataset, which spans from 2000 to 2019, nine out of twelve countries show negative Backus-Smith correlations, with an average of -0.06. This is still negative and near zero,

strong negative correlation with Δc while Δq^T and Δq^{NT} have positive or close to zero correlations, giving us a clue to the composition of the Backus-Smith correlation.

Table 3: Backus-Smith Correlations

	$Corr(\Delta c, \Delta q)$	$Corr(\Delta c, \Delta q^T)$	$Corr(\Delta c, \Delta q^{NT})$	$Corr(\Delta c, \Delta q^R)$
Austria	-0.066	-0.031	0.131	-0.489
Belgium	-0.029	0.047	-0.087	-0.118
Finland	0.246	0.481	-0.027	-0.020
France	0.307	0.219	0.467	-0.162
Germany	-0.205	-0.012	-0.122	-0.551
Greece	-0.075	0.090	-0.110	-0.080
Ireland	-0.418	-0.218	-0.242	-0.541
Italy	0.135	0.048	0.288	0.011
Luxembourg	-0.082	0.302	-0.159	-0.260
Netherlands	-0.039	-0.149	0.176	-0.299
Portugal	-0.275	-0.183	-0.137	0.052
Spain	-0.203	0.114	-0.272	-0.235
Average	-0.059	0.059	0.008	-0.224

$q = ln(P_{EU15t}/P_{it})$, $q^T = ln(P^T_{EU15t}/P^T_{it})$, $q^{NT} = ln(P^{NT}_{EU15t}/P^{NT}_{it})$, $q^R = ln(P^R_{EU15t}/P^R_{it})$ where P_{EU15t} is the price level of 15 European countries' average. $c = ln(C_{it}/C_{EU12t}), y = ln(Y_{it}/Y_{EU12t})$ where C_{EU12t}, Y_{EU12t} are geometric averages of C, Y over 12 eurozone countries. Data period is from 2000 to 2019 and data are at annual frequency. Results from using consumption per capita are in the online appendix.

However, again, this is not a systematic way to examine the correlation between relative consumption and the real exchange rate. As I did for the Balassa-Samuleson effect, I run the four regressions to examine the relationship between changes in aggregate and sectoral real exchange rates (q, q^T, q^{NT}, q^R) and real relative consumption (c). Table 4 reports the results. Unlike the case of the Balassa-Samuelson effect, now the cross-sectional regressions do not show any significant results. However, regressions that capture time-series variations show significant results. In the time-series dimension, when a country's aggregate consumption grows 1% more than the EU 12 average, the country's price level gets 0.14% more expensive than that of other countries. In addition, q^R is the one that has significant β^R estimates in both regressions, and the estimated sizes are very large.

Given that the time-series pattern is stronger, I conduct regression-based decomposition again for two regressions for time-series variations. The right panel in Figure 4 shows the results. In both cases, while the tradable real exchange rate component shows positive correlations, nontradables and the rent real exchange rate show negative correlations. In particular, the red bar is the biggest even though the γ^R is only 0.16. This shows how important the rent real exchange rate is in understanding the Backus-Smith correlation.

being starkly different from 1. This suggests that while the nominal exchange rate plays a significant role, there are other mechanisms at work that contribute to the negative Backus-Smith correlation.

Table 4: Backus-Smith Correlation Regressions

		Cross-section						Time-series			
		$\bar{q}$	$\bar{q}^T$	$\bar{q}^{NT}$	$\bar{q}^R$			Δq	Δq^T	Δq^{NT}	Δq^R
Country Average	$\bar{c}$	0.03	0.00	0.03	0.07	Growth Rate	Δc	-0.14**	0.02	-0.15***	-0.53***
		(0.02)	(0.02)	(0.02)	(0.05)			(0.07)	(0.05)	(0.06)	(0.23)
	R^2	0.07	0.01	0.07	0.09		R^2	0.03	0.00	0.01	0.06
	N	12	12	12	12		N	240	240	240	240
		q	q^T	q^{NT}	q^R			q	q^T	q^{NT}	q^R
Time Fixed Effect	c	0.02	0.00	0.03	0.07	Country Fixed Effect	c	-0.16**	0.10*	-0.21	-0.72**
		(0.02)	(0.02)	(0.02)	(0.06)			(0.07)	(0.06)	(0.14)	(0.37)
	R^2	0.07	0.01	0.06	0.09		R^2	0.09	0.05	0.06	0.17
	N	240	240	240	240		N	240	240	240	240

$q = ln(P_{EU15t}/P_{it})$, $q^T = ln(P^T_{EU15t}/P^T_{it})$, $q^{NT} = ln(P^{NT}_{EU15t}/P^{NT}_{it})$, $q^R = ln(P^R_{EU15t}/P^R_{it})$ where P_{EU15t} is the price level of 15 European countries' average. $c = ln(C_{it}/C_{EU12t}), y = ln(Y_{it}/Y_{EU12t})$ where C_{EU12t}, Y_{EU12t} are geometric averages of C, Y over 12 eurozone countries. Data period is from 2000 to 2019 and data are at annual frequency. All standard errors are computed using a panel-corrected standard errors method under the assumption of period correlation (cross-sectional clustering). Parentheses below estimates include standard deviations. * means 10% significance, ** means 5% significance, *** means 1% significance.

3 Model: Inelastic Housing Supply and Real Exchange Rates

Motivated by the empirical evidence suggesting the crucial role of housing rent, to explore its role in international business cycles I construct a standard two-country DSGE model and extend it in several dimensions. First, I assume that both the home and foreign country have tradable, nontradable, and construction sectors. In addition, there is a distribution margin for consuming retail tradable goods, as in Berka et al. (2018). Second, I incorporate housing in the model following Davis and Heathcote (2005). Each country needs to accumulate housing stock so they can obtain housing services from it. Importantly, to produce houses, they need to use residential-zoned land as a production input, which is under fixed supply. In addition, it takes one period to build houses. Last, I assume an incomplete market between two countries so the two countries can insure their risks only through noncontingent bonds, as in Corsetti et al. (2008). Because I use data of primarily eurozone countries, which use the same currency (euro), I have not modeled any monetary component. I present only the home country, and the foreign economy is the same as the home country is because the model has a symmetric structure.

Households The home country's infinitely lived representative household maximizes the lifetime utility defined as

$$U = \sum_{t=0}^{\infty} E_t[\beta^t(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi})], \quad \beta < 1, \tag{5}$$

where C_t is the aggregate consumption bundle and N_t is the home labor supply. Note that the labor disutility function is separable from the consumption utility function.[10]

[10]While many papers studying real exchange rate dynamics use a separable utility like my model (e.g., Chari et al. (2002)), other papers use a non-separable utility function (e.g., Karabarbounis (2014)). They generate the negative Backus-Smith correlation even under a complete market because non-

When optimizing this lifetime utility, the representative household faces the following budget constraints:

$$s.t.\quad P_tC_t + D_{t+1}/R_{t+1} + P_{RI,t}I_{RI,t} = W_tN_t + P_{R,t}H_{t-1} + P_{l,t}l_t + D_t - \frac{\phi^c}{2}D_{t+1}^2, \tag{6}$$

$$H_t = (1-\delta^H)H_{t-1} + I_{RI,t}. \tag{7}$$

P_t means the price of the aggregate consumption bundle, which can be interpreted as an aggregate price index. Households can save by investing in the international bond market. D_{t+1} represents the amount of bonds purchased by the household, where R_{t+1} is the return on it. $I_{RI,t}$ represents new housing construction, and it enters into the law of motion of housing capital. H_t denotes the housing capital that will be available in time $t+1$. W_t represents the wage earned by supplying labor, H_{t-1} means the housing stock they currently have, $P_{R,t}$ is the housing rent, and l_t and $P_{l,t}$ represent the residential-zoned land supply by household and its price.[11] Lastly, when households save and borrow through international bonds, there is a convex cost associated with owning them, denoted as $\frac{\phi^c}{2}D_{t+1}^2$. This method is one suggested by Schmitt-Grohé and Uribe (2003) to guarantee a unique steady state in a two-country model under an incomplete market.

The aggregate consumption bundle is defined as the CES aggregation of a housing service $(C_{R,t})$ and non-housing consumption bundle $(C_{NR,t})$, as in equation (8).

$$C_t = (\gamma_R^{\frac{1}{v}} C_{R,t}^{1-\frac{1}{v}} + (1-\gamma_R)^{\frac{1}{v}} C_{NR,t}^{1-\frac{1}{v}})^{\frac{v}{v-1}}. \tag{8}$$

Housing services are assumed to be proportional to housing stock (H_t), implying that $C_{R,t} = H_{t-1}$. v is the elasticity of substitution between housing services and non-housing consumption, and γ_R is the relative weight of housing services.

The non-housing consumption bundle is defined over a tradable consumption bundle $(C_{T,t})$ and nontradable consumption bundle $(C_{NT,t})$, as in equation (9).

$$C_{NR,t} = ((1-\gamma_{NT})^{\frac{1}{\theta}} C_{T,t}^{1-\frac{1}{\theta}} + \gamma_{NT}^{\frac{1}{\theta}} C_{NT,t}^{1-\frac{1}{\theta}})^{\frac{\theta}{\theta-1}}. \tag{9}$$

θ is the elasticity of substitution between a tradable consumption bundle $(C_{T,t})$ and nontradable consumption $(C_{NT,t})$ and γ_{NT} is the relative weight of nontradable consumption. A tradable consumption has additional layers, as in equation (10):

$$C_{T,t} = (\omega_H^{\frac{1}{\lambda}} C_{H,t}^{1-\frac{1}{\lambda}} + (1-\omega_H)^{\frac{1}{\lambda}} C_{F,t}^{1-\frac{1}{\lambda}})^{\frac{\lambda}{\lambda-1}}. \tag{10}$$

separability breaks the one-to-one relationship between relative price and consumption under a complete market, whereby leisure also affects marginal utilities.

[11] Following other research (Davis and Heathcote 2005, Kaplan et al. 2020), we assume that the government assigns a certain amount of land as residential-zoned land every period.

$C_{T,t}$ is defined as an aggregation of a (retail) home-tradable consumption bundle ($C_{H,t}$) and a (retail) foreign-tradable consumption bundle ($C_{F,t}$). ω_H is the relative weight of $C_{H,t}$, and ω_H larger than 0.5 implies home bias. λ is the elasticity of substitution between the home tradable and foreign tradable consumption bundles.

Both (retail) home and foreign-tradable consumption bundles are defined as the aggregation between each wholesale tradable good ($X_{H,t}$, $X_{F,t}$) and distribution margin services ($V_{H,t}$, $V_{F,t}$), as in equation (11) and equation (12).

$$C_{H,t} = ((1-\chi_{NT})^{\frac{1}{\phi}} X_{H,t}^{1-\frac{1}{\phi}} + \chi_{NT}^{\frac{1}{\phi}} V_{H,t}^{1-\frac{1}{\phi}})^{\frac{\phi}{\phi-1}}, \tag{11}$$

$$C_{F,t} = ((1-\chi_{NT})^{\frac{1}{\phi}} X_{F,t}^{1-\frac{1}{\phi}} + \chi_{NT}^{\frac{1}{\phi}} V_{F,t}^{1-\frac{1}{\phi}})^{\frac{\phi}{\phi-1}}. \tag{12}$$

In other words, to consume the traded goods, households must use nontradable services. This distribution margin is justified by the distribution cost incurred by local input, such as labor for transporting goods. $\chi_{NT,t}$ defines the relative importance of the distribution margin, and ϕ is the elasticity of substitution between tradable goods and the distribution margin. This consumption structure implies the aggregate price index P_t and non-housing consumption price index $P_{NR,t}$, as in equation (13) and equation (14).

$$P_t = (\gamma_R P_{R,t}^{1-v} + (1-\gamma_R) P_{NR,t}^{1-v})^{\frac{1}{1-v}}, \tag{13}$$

$$P_{NR,t} = ((1-\gamma_{NT}) P_{T,t}^{1-\theta} + \gamma_{NT} P_{NT,t}^{1-\theta})^{\frac{1}{1-\theta}}. \tag{14}$$

Note that $P_{R,t}$ is housing rent, my major focus. Equation (13) and equation (14) imply that the aggregate price level is a weighted average of the price of tradables, the price of nontradables, and housing rent.

The tradable consumption bundle price index $P_{T,t}$ is defined as in equation (15), while the non-tradable consumption bundle price $P_{NT,t}$ is the price of nontradables.

$$P_{T,t} = (\omega_H \tilde{P}_{H,t}^{1-\lambda} + (1-\omega_H) \tilde{P}_{F,t}^{1-\lambda})^{\frac{1}{1-\lambda}}. \tag{15}$$

Because I assume the presence of the distribution margin, I know that the retail price of home tradable $\tilde{P}_{H,t}$ and foreign tradable $\tilde{P}_{F,t}$ should contain distribution margins. These are well denoted in equations (16) and (17).[12]

$$\tilde{P}_{H,t} = ((1-\chi_{NT}) P_{H,t}^{1-\phi} + \chi_{NT} P_{NT,t}^{1-\phi})^{\frac{1}{1-\phi}}, \tag{16}$$

$$\tilde{P}_{F,t} = ((1-\chi_{NT}) P_{F,t}^{1-\phi} + \chi_{NT} P_{NT,t}^{1-\phi})^{\frac{1}{1-\phi}}. \tag{17}$$

[12]This implies that not only the terms of trade ($\frac{P_t^F}{P_t^H}$) but also the nontradable real exchange rate ($q_t^{NT} = \frac{P_{N,t}^*}{P_{N,t}}$) affects the tradable real exchange rate (q_t^T).

These price indices are combined to generate sectoral real exchange rates as follows:

$$Q_t = \frac{P_t^*}{P_t} \quad (q_t = log(Q_t)), \tag{18}$$

$$Q_t^i = \frac{P_{i,t}^*}{P_{i,t}} \quad (q_t^i = log(Q_t^i)) \text{ for } i \in \{T, NT, R\} \tag{19}$$

International Asset Market I assume an incomplete market so that both countries' households can insure themselves against the shock only via noncontingent bonds. As famously noted by Corsetti et al. (2008), introducing an incomplete market generates wealth effects from the tradable sector productivity shock. Among the methods for ensuring a stationary equilibrium in a two-country incomplete market model suggested by Schmitt-Grohé and Uribe (2003), I chose the convex portfolio adjustment cost. Assuming symmetric economies, the optimality condition for international saving and borrowing D_t, D_t^* is as in equation (20).

$$R_t = E_t[\frac{1}{\beta}(\frac{C_t^{-\sigma}}{C_{t+1}^{-\sigma}})(\frac{P_{t+1}}{P_t})(\phi^c D_{t+1} + 1)] = E_t^*[\frac{1}{\beta^*}(\frac{(C_t^*)^{-\sigma}}{(C_{t+1}^*)^{-\sigma}})(\frac{P_{t+1}^*}{P_t^*})(\phi^c D_{t+1}^* + 1)]. \tag{20}$$

Once I ignore the ϕ^c, which will be calibrated as tiny, I see that the relationship between relative consumption and the real exchange rate holds under expectation, not state by state. This allows the model to deviate from perfect risk sharing and provides an environment for generating the negative Backus-Smith correlation.

Intermediate Good Production Moving toward to the production side, I have three sectors—tradable, nontradable, and construction as follows:

$$Y_{i,t} = A_{i,t} N_{i,t}^{\alpha^i} \text{ for } i \in \{H, N, CR\} \tag{21}$$

There are no adjustment costs for labor reallocation, and I assume there is no non-housing capital for brevity. The foreign country has a symmetric production structure. Each sector has its own productivity. I assume they are AR(1) processes, as follows:

$$ln(A_{i,t}) = ln(\bar{A}_i) + \rho_H(ln(A_{i,t-1}) - ln(\bar{A}_i)) + \epsilon_{i,t} \text{ for } i \in (H, N, CR). \tag{22}$$

Housing Construction To construct new houses ($I_{RI,t}$), real estate developers in each country combine land and construction goods. τ implies the share of residential-zoned land for the housing production.

$$I_{RI,t} = Y_{CR,t}^{1-\tau} l_t^{\tau}. \tag{23}$$

The law of motion for housing stock is stated in equation (24). As is clear in the law of motion, it takes one period to build new houses, and new houses become available for consumption only after one period. In addition, housings depreciate by δ^H.

$$H_{t+1} = (1 - \delta^H)H_t + I_{RI,t}. \tag{24}$$

In my model, $Y_{CR,t}$ is effectively the residential investment, which does not include the land component.[13] In addition, I focus only on residential buildings because I cannot observe commercial rents in the data. The construction sector in my model is a residential building construction sector.

I assume that land is supplied in the fixed amount every period. Following Davis and Heathcote (2005), I do not attempt to model the supply of residential-zoned land, which requires consideration of infrastructure development and the zoning process. I assume that through deconstruction of existing buildings and the government's new zoning assignment, a constant amount of residential zoned land is supplied.

$$l_t = \bar{l}. \tag{25}$$

The Role of Land in Housing Supply Elasticity The role of land in housing production is easy to see when the price of construction goods is fixed. If $P_t^{CR} = \bar{P}^{CR}$ and $l = \bar{l}$, the real estate developer's first-order conditions imply the following:

$$Y_t^{CR} = (\frac{\bar{P}^{CR}}{l^\tau(1-\tau)P_t^{RI}})^{-\frac{1}{\tau}}. \tag{26}$$

Substituting this in the production function of the real estate developer, I can calculate the housing supply function and supply elasticity as below.

$$I_t^H = (P_t^H)^{\frac{1-\tau}{\tau}}(1-\tau)^{\frac{1-\tau}{\tau}}(\bar{P}^{CR})^{\frac{\tau-1}{\tau}}\bar{l}. \tag{27}$$

$$\frac{\partial ln(I^H)}{\partial ln(P^H)} = \frac{1-\tau}{\tau}. \tag{28}$$

This implies that the larger the land input share, the more housing supply elasticity decreases, which implies a steeper supply curve. Note that this is the supply for the new housing flow, not the total housing service supply. For the housing service supply, in the short run, the elasticity is 0 because it takes one period to build a house. In addition, depending on δ^H, new housing flows might be very small compared to the total housing stock, which makes aggregate housing service supply more inelastic.

[13] Davis and Heathcote (2005) allow the real estate developer to combine manufacturing goods, services, and construction goods to generate residential investment. On the other hand, Kaplan et al. (2020) model a housing sector in which households combine lands and labor to produce housing.

Relative Output and Relative Consumption To study the Balassa-Samuelson effect and the Backus-Smith correlation, I need to define the relative real output per capita (y_t) and relative real consumption growth (Δc_t) in the model. First, I define output (per capita) (Y_t) as follows:[14]

$$Y_t = P_t C_t + P_{RI,t} I_{RI,t} + P_{H,t}(Y_{H,t} - C_{H,t}) - P_{F,t}(Y^*_{F,t} - C_{F,t}). \tag{29}$$

Then, I construct relative output per capita y as follows:

$$y_t = ln(Y_t) - ln(Y^*_t). \tag{30}$$

Also, I use the home country aggregate consumption bundle as a numeraire by normalizing $P_t = 1$. For the relative consumption growth (Δc_t), I use each country's aggregate consumption as follows.

$$\Delta c_t = \Delta(ln(C_t) - ln(C^*_t)). \tag{31}$$

Equilibrium of the Model and Solution Methods Since the equilibrium definition of my model is very standard, for the sake of brevity I skip the definition of model equilibrium. The model is solved using the first-order approximation with Dynare.

4 Quantitative Analysis: Model Simulation

In this section, I simulate our structural model to provide a more detailed quantitative analysis of the relationship between housing and the real exchange rate. A proper examination of the relationship between housing and the real exchange rate requires a general equilibrium perspective. My simulation procedure closely follows the strategy of Berka et al. (2018). Although my model has only two countries, I can map the simulated data onto the actual data by treating the model home country as the relevant EU country, and assuming the model foreign country as the EU average. This gives me simulated panel data on 8 countries for a 20-year period.[15] Using these simulated data, I replicate the empirical analysis I did in the earlier section and explore the role of the housing sector in real exchange rate determination.

Model Calibration To render my simulation analysis quantitatively realistic, proper calibration is required. My calibration strategy aims to match housing-related moments, and productivity shock processes are directly calibrated from data. Empirical

[14]Note that $P_{H,t} = P^*_{H,t}$ and $P_{F,t} = P^*_{F,t}$. In my model, I do not assume any other frictions such as variable mark-up. The law of one price holds for every good in the model.

[15]For the model simulation, I use only eight eurozone countries, which provide the industry-level productivity data in EUKLEMS 2023 for productivity shock process calibration. These are Austria, Belgium, Spain, Finland, France, Germany, Italy, and the Netherlands.

moments of the real exchange rates are not targeted in my calibration.

Table 5: Model Calibration

Parameters	Variable	Value	Reference
1. Non-Housing Parameters			
Household			
Discount factor, yearly	β	0.99	
Relative risk aversion	σ	2	
Macro Frisch elasticity	ψ	1	
Non-Residential Consumption Aggregator			
Non-Tradable weight	γ^{NT}	0.4	Berka et al. (2018)
ES between traded and non-traded	θ	0.7	Berka et al. (2018)
Tradable Consumption Aggregator			
Home-bias	ω^{H}	0.5	No Homebias
ES between retail H and F	λ	8	Corsetti et al. (2010)
Distribution Margin			
Distribution Margin Weight	χ^{NT}	0.32	Goldberg and Campa (2010)
ES betwen retail and distribution service	ϕ	0.25	Berka et al. (2018)
Production			
Elasticity of Labor	α	1	Berka et al. (2018)
International Financial Market			
Portfolio Adjustment Cost	ϕ^{C}	0.001	Benigno and Thoenissen (2008)
2. Housing Parameters			
Residential Consumption			
Housing Service Weight	γ^{R}	0.25	
ES between housing and non-housing	v	0.85	Davidoff and Yoshida (2013)
Residential Building Production			
Land Input Share	τ	0.35	Combes et al. (2021)
Depreciation Rate of Housing	δ^{H}	0.0025	

Non-housing Parameters The upper panel of Table 5 shows my calibration for non-housing parameters. I use $\beta = 0.99$, assuming quarterly frequency in the model, matching the long-run real interest rate among eurozone countries. For the coefficient of relative risk aversion and Frisch elasticity of labor supply, I set $\sigma = 2$ and $\psi = 1$, which are standard values used in DSGE modeling. For the relative weight between the tradable and nontradable, I set $\gamma^{NT} = 0.4$ to match the expenditure shares of each in the data. The elasticity of substitution between the tradable and the nontradable is set as $\theta = 0.7$, following Berka et al. (2018). Given the presence of a distribution margin that generates home bias by itself and the homogeneity of eurozone countries, I set $\omega^{H} = 0.5$, which implies no home bias at the retail level. Elasticity between the home tradable and the foreign tradable (which is also called trade elasticity) is set as $\lambda = 8$, following Berka et al. (2018). Trade elasticity has been known to be small in the short run, lower than 1, and large in long run, at higher than 1.[16] Because my trade elasticity, λ, is the elasticity between the retail home and foreign goods and both contain the domestic distribution margin, it is not exactly the same as

[16] Corsetti et al. (2008) use 0.5 for their first case and 4 for their second case. Cross-country estimates imply elasticity larger than 1 (Broda and Weinstein 2006), while the time-series estimates based on the response of import quantities to the exchange rate suggest elasticity less than 1 (Feenstra et al. 2018, Amiti et al. 2022)

the trade elasticities used in other research.[17] For weights for the distribution margin, I use the estimates of Goldberg and Campa (2010) and calculate the average of eight eurozone countries' distribution margins for household consumption, which implies $\chi^{NT} = 0.32$. Lastly, regarding the portfolio adjustment cost, following Benigno and Thoenissen (2008), I set ϕ^C equal to 0.001.[18]

Housing Parameters The lower panel of Table 5 shows my calibration for housing parameters. Calibrating housing-related parameters is difficult, and papers in the literature use different values. These parameters include the weight of housing service consumption (γ^R), the elasticity of substitution between residential consumption and non-housing consumption (v), the land input share in the housing production function (τ), and the housing depreciation rate (δ^H).

Several papers provide information on these parameters. For example, Combes et al. (2021) show that τ in France is roughly 35% using detailed French housing construction data, which include house prices, land sizes, and land prices. However, papers that study the US housing market, such as Kaplan et al. (2020) and Favilukis et al. (2012), use 0.25 and 0.1 for τ. Also, for v, Davidoff and Yoshida (2013) suggest that the range should be between 0.4 and 0.9 using aggregate time-series data under a non-homothetic preference assumption. However, this is contradicted by Davis and Ortalo-Magne (2021), who show that the housing rent expenditure share is constant across regions over time and suggest using Cobb-Douglas specifications. Given the absence of consensus on these parameters, I target five empirical moments related to the housing sector of eight eurozone countries: (1) value of residential structure capital stock over GDP ($RCOY$), (2) residential investment over GDP ($RIOY$), (3) share of construction sector labor over total labor ($NConRatio$), (4) household rent expenditure share (REW), and (5) new housing flow over housing stock ($HFoHS$).

I come up with the model counterparts for those empirical moments as follows. First, for the residential structure capital over GDP ($RCOY$), I define the net stock of residential structure, S, as follows under the assumption that residential structure depreciates by δ^S per period. Under the steady state, I can define S as follows:[19]

$$S = \sum_{k=1}^{\infty} (1 - \delta^S)^k Y_{CR_{t-k}}. \tag{32}$$

In the steady state, $P_{CR}S = \frac{P_{CR}Y_{CR}}{\delta^S}$ and $P_{RI}H = \frac{P_{RI}I_{RI}}{\delta^H}$ hold from the law of motion for housing stock and residential capital structure. Also, $P_{RI}I_{RI} = \frac{P_{CR}Y_{CR}}{(1-\tau)}$ holds from

[17] Corsetti et al. (2010) show that this implies a lower elasticity of substitution between traded wholesale goods, due to the presence of distribution services.

[18] Rabanal and Tuesta (2010) provide estimates of 0.007 for quarterly data, but there is no big difference in model-simulated results even though I use their value.

[19] Land is not included in residential capital stock in the national accounting system.

the optimal condition of the real estate developer. Combining all these, under the steady state, the following holds:[20]

$$\frac{P_{CR}S}{P_{RI}H} = \frac{(P_{CR}Y_{CR})/\delta^S}{P_{CR}Y_{CR}/((1-\tau)*\delta^H)} = \frac{(1-\tau)*\delta^H}{\delta^S} = \frac{(1-\tau)*(1-(1-\delta^S)^{1-\tau})}{\delta^S}. \quad (33)$$

Consequently, residential structure capital stock over GDP $(RCOY)$ in my model will be defined as

$$RCOY = \frac{(1-\tau)*(1-(1-\delta^S)^{1-\tau})}{\delta^S}\frac{P_{RI}H}{PY}. \quad (34)$$

I define the residential investment over GDP $(RIOY)$ as follows:

$$RIOY = \frac{P_{CR}Y_{CR}}{PY}, \quad (35)$$

The share of construction sector labor $(NConRatio)$ is defined as

$$NConRatio = \frac{N_{CR}}{N_H + N_N + N_{CR}}. \quad (36)$$

It is especially important to match this moment because it determines the size of the effect of the construction sector productivity shock on the aggregate economy via the labor market in the model.

The household expenditure weight on housing rents (REW) is defined as follows:

$$REW = \frac{P_R C_R}{PC} = \frac{P_R H}{PC}, \quad (37)$$

Lastly, I define new housing flow over housing stock ($HFoHS$) as follows:

$$HFoHS = I_H/H. \quad (38)$$

To replicate these moments, I set $\gamma^R = 0.25$. In addition, I set elasticity of substitution between housing and non-housing consumption as $v = 0.85$. This is also chosen to match the increasing patterns of rent expenditure weights over time in all eurozone countries in Figure 1. The land input share in housing production is set as $\tau = 0.35$ following Combes et al. (2021). This is much larger than the values used by Kaplan et al. (2020) or Favilukis et al. (2012), both of which study the US housing market. However, eurozone countries have substantially lower housing supply elasticities compared with that of the US. In addition, while there is some heterogeneity across eurozone countries' housing supply elasticities, a recent estimate

[20] As explained by Davis and Heathcote (2005), under our Cobb-Douglas housing production function using land and residential structure with τ land input share, $1-\delta^H = (1-\delta^S)^{1-\tau}$.

suggests that France's housing supply elasticity is in the middle among the eurozone countries (Caldera and Åsa Johansson 2013). For the housing depreciation rate, I use $\delta^H = 0.0025$, implying 1% annual depreciation.

Table 6: Housing Sector Moments: Data vs Model Steady State

	Data	Model
Supply side		
Residential Capital over GDP ($RCOY$)	1.457	1.403
Residential Investment over GDP ($RIOY$)	0.029	0.021
Labor Share of Construction Sector ($NConRatio$)	0.017	0.025
Housing Flow over Housing Stock ($HFoHS$)	0.009	0.009
Demand side		
Housing Rent Expenditure Share (REW)	0.161 (0.212)	0.170
Tradable Expenditure Share	0.516	0.497
Nontradable Expenditure Share	0.328	0.331

Data period for 8 Eurozone countries is (2000-2019). Note that the construction sector in my model is effectively the residential construction sector, not the total construction sector. According to the European Construction Industry Federation, 50.4% of total construction is estimated to be residential construction in 2022. So, I use half of the value of the corresponding construction sector for construction sector-related variables when I match the empirical moments of the construction sector in my model.

Table 6 shows how my model performs in terms of replicating these moments of average of eurozone countries in the steady state. The model successfully replicates most of the moments. In particular, it replicates the fact that housing is very inelastically supplied ($HFoHS$), housing rent accounts for a substantial portion of the aggregate expenditure (REW), and the residential construction sector accounts for a small portion of the aggregate labor market ($NConRatio$).

Sectoral Productivity Shocks In my simulation, I use sectoral productivity shocks as the main drivers of international business cycles. To estimate each sector's productivity, I closely follow the estimation procedure used by Berka et al. (2018) and extend their estimates up to 2019. The online appendix details the procedure I used to estimate these processes. Here, I provide a brief description.

To estimate sectoral productivity shock processes, I use the GGDC 1997 database and EUKLEMS 2023. GGDC 1997 provides all industries' productivity levels.[21] Then, for each country, by dividing each industry's productivity by that of the geometric average of the eurozone,[22] I calculate each industry's relative productivity level against that of the eurozone average. Using the EUKLEMS 2023 database, which provides the industry-level growth rate for each country, I calculate the productivity growth rate

[21] This is a given industry's productivity relative to that industry in the US. I cancel out the US component, by dividing productivities with the eurozone average (relative to the US).

[22] I use the eleven European countries that provide the productivity data in EUKLEMS 2023, three of which, Sweden, Denmark, and UK, are not eurozone members. I included them because they show interesting patterns of sectoral productivity distribution, which will be explored in detail in the later part of the paper. Because those three countries are only used for calculating average, overall simulation results do not differ even though I calculate the eurozone average only with the remaining eight countries.

of each industry relative to that of eurozone average.

By combining these relative levels and growth rates, I construct panel data for each industry's relative productivity in each country. Lastly, by classifying these industries into tradable, nontradable, and construction sectors, and by averaging with the value-added of each industry as a weight, I obtain panel data for all sectoral productivity levels relative to the eurozone average.[23]

With this panel data, I estimate sectoral productivity shock processes for each country. Thus, what I estimate are the following relative sectoral productivity processes:

$$\alpha_{Y,it} - \bar{\alpha}_Y = \rho_H(\alpha_{Y,it-1} - \bar{\alpha}_Y) + \epsilon_{Y,it} \text{ for } Y \in \{H, N, CR\}.$$

where $\alpha_{Y,jt} = ln(\frac{A_{Y,jt}}{A_{Y,EUt}})$ for $Y \in \{H, N, CR\}$. Data are from 2000 to 2019 at annual frequency. Note that while my data are at annual frequency, the model simulation will be conducted at quarterly frequency, so I convert estimated parameters to quarterly frequencies by taking the quadratic root of ρ. Also, the variance-covariance matrix of the shock processes is estimated under the assumption that the shock is i.i.d at quarterly frequency. In addition, I allow covariance relationships among the productivity processes of all sectors and countries. Table 7 reports the results of the estimation.

Table 7: Estimated Sectoral TFP Processes

	A. Cross-section			B. Time-series					
	Mean values			AR(1) Coefficients			Standard Deviations		
	$\bar{a}_H$	$\bar{a}_N$	$\bar{a}_{CR}$	ρ_H	ρ_N	ρ_{CR}	σ_H	σ_N	σ_{CR}
AUT	-0.241	-0.118	0.119	0.918	0.894	0.966	2.367	0.936	2.344
BEL	0.135	0.011	0.205	0.983	0.976	0.971	2.700	0.907	2.017
ESP	-0.018	-0.132	-0.172	0.873	0.987	0.945	2.409	0.951	3.499
FIN	-0.080	-0.060	0.231	0.939	0.769	0.946	6.198	1.262	2.804
FRA	0.040	-0.046	-0.139	0.925	0.997	0.989	2.716	0.583	1.862
GER	-0.034	0.046	-0.080	0.973	0.905	0.962	2.198	1.206	2.228
ITA	-0.106	-0.036	-0.003	0.951	0.959	0.987	1.326	0.708	2.402
NLD	0.264	0.145	-0.080	0.990	0.986	0.986	2.919	1.153	3.359
AVG	-0.005	-0.024	0.010	0.944	0.934	0.969	2.854	0.963	2.564

Several interesting patterns emerge. First, as the productivity itself is defined as relative productivity, it is natural that the cross-country averages of $\alpha_H, \alpha_N, \alpha_{CR}$ are close to 0. Also, the relative productivities of both the tradable sector and the construction sector show much larger cross-sectional and time-series variations compared with those of the nontradable sector. This aligns with estimates from previous research.

Simulation Procedure Given the calibrated sectoral productivity shock processes

[23]The industry classifications of EUKLEMS are a bit different from those of the GGDC 1997 database. However, they are closely related. Consequently, as I explain in detail in the online appendix, I generated a sectoral concordance table and use that accordingly. I have a total 12 tradable industries, 9 nontradable industries, and 1 construction industry. Graphs of these estimated relative sectoral productivities are presented in the online appendix.

and the model calibration explained above, I simulate the eight countries whose productivity data are available. The simulated periods are 80 quarters, as in the data (2000-2019). In my simulation, the core assumption is that each of these eight countries is a home country and the eurozone average is a foreign country. Under this assumption, because the sectoral productivity shock processes are estimated in units of each country's sectoral productivity relative to that of the eurozone average, the simulated shocks will be fed only into the home country, while the foreign country does not receive any shocks during the simulation. Only the transmissions of home country shocks affect the foreign country. After each simulation, I collect only the home country's aggregate real exchange rates, sectoral real exchange rates, relative real GDP per capita, and relative real consumption. This gives me simulated panel on such variables, and this simulated panel is comparable to what I have in actual data.

Given the panel data from each simulation, I replicate the empirical analysis that I performed in the empirical section, and I repeat the whole procedure 500 times. This repetition gives me the distributions of the parameters of interest, such as cross-sectional and time-series variations of the real exchange rates, Balassa-Samuelson regression coefficients, and Backus-Smith regression coefficients. Such distributions will be compared with actual data estimates. Also, this procedure will be repeated in a different calibration environment to understand how the housing market affects real exchange rate dynamics.

One thing to note is that during this simulation, none of the empirical moments of the real exchange rate were targeted. This simulation exercise should be understood as exploring how far we can go in explaining the role of housing rent in the real exchange rate by combining the standard model of the housing market and the two-country international business cycle model given the productivity processes externally calibrated from the productivity dataset.

4.1 Simulation Result: Model-generated Real Exchange Rates

In this subsection, I provide simulation results that offer insights into how the housing sector affects the properties of real exchange rates. I present results for cross-section (cross-country) and time-series variations of the real exchange rate first, then for the Balassa-Samuleson effect, and finally the Backus-Smith correlation.

4.1.1 Housing and Variations of Real Exchange Rates

Table 8 compares cross-section and time-series variations of model-generated real exchange rates under different calibrations with those of the data. The upper panel is about cross-sectional variations, and the lower panel shows time-series variations.

Data and the Baseline Model Column (1) shows the variations of the real exchange

Table 8: Simulated Cross-sectional and Time-series Variations of RER

	(1) Data	(2) Bond Baseline	(3) Arrow Debereu Baseline	(4) Bond ($\tau = 0.01$)	(5) Bond ($\delta^S = 0.99$)	(6) Bond (τ=0.01, δ^S=0.99)	(7) Bond ($\bar{A}_j^{CR}/\bar{A}_{EU}^{CR} = 1$)
Cross-section							
$\sigma_j(q_{jt})$	0.121	0.086	0.053	0.088	0.093	0.099	0.080
$\sigma_j(q_{jt}^T)$	0.081	0.039	0.029	0.039	0.039	0.039	0.040
$\sigma_j(q_{jt}^{NT})$	0.149	0.121	0.088	0.120	0.120	0.119	0.121
$\sigma_j(q_{jt}^R)$	0.297	0.198	0.134	0.249	0.210	0.257	0.144
Time-series							
$\sigma_t(q_{jt})$	0.025	0.033	0.022	0.034	0.034	0.037	0.033
$\sigma_t(q_{jt}^T)$	0.022	0.018	0.013	0.017	0.017	0.017	0.017
$\sigma_t(q_{jt}^{NT})$	0.039	0.054	0.041	0.053	0.052	0.052	0.054
$\sigma_t(q_{jt}^R)$	0.072	0.038	0.009	0.066	0.061	0.084	0.038

The first column shows actual standard deviations calculated from the data. From the second to the last column, each column contains the means of simulation-generated standard deviations of 500 simulations. The second column is the simulation result under the baseline calibration in Table 5 and incomplete market assumption, while the other columns show the results with the changes of some parameters or risk-sharing assumptions as specified in the first row. Other than the specified changes in the first row, the other parameters are always as shown in Table 5.

rate in the data, and column (2) shows model-simulated variations of the real exchange rate under the baseline calibration in Table 5. The upper panel shows that our baseline model generates substantial cross-sectional variations. In addition, the relative sizes of variations of sectoral real exchange rates are also consistent with the data, showing the largest variations in the rent real exchange rate. Moving to the lower panel, the baseline model also generates substantial time-series variations comparable to the data. One difference with data is that our model generates larger variations in the nontradable real exchange rate than that of rent real exchange rate.

Role of a Wealth Effect First, a complete market is assumed instead of an incomplete market to understand its mechanics. An incomplete market is known to generate a deviation from perfect risk-sharing, which creates room for the wealth effect. Under an incomplete market, if the home country receives a positive tradable sector productivity shock and gets wealthier than the foreign country, its aggregate demand increases more than that of the foreign country (Corsetti et al. 2008.) This demand differential will increase consumption and price more than in the foreign country, resulting in real appreciation. Column (3) shows the result of shutting down the wealth effect. Compared with column (2), all variations decrease significantly. This implies that the wealth effect boosts variations in all dimensions. The most striking change comes from the time-series variation of the rent real exchange rate, $\sigma_t(q_{jt}^R)$, which drops from 0.038 to 0.009. Such a drop is much more significant than other sectoral real exchange rates. This shows the importance of inelastic housing supply.

Figure 5 explains why inelastic supply generates such a pattern. As in the left graph of Figure 5, the wealth effect shifts the aggregate demands and generates changes in the prices of nontradable services. However, as in the right graph, such an effect is

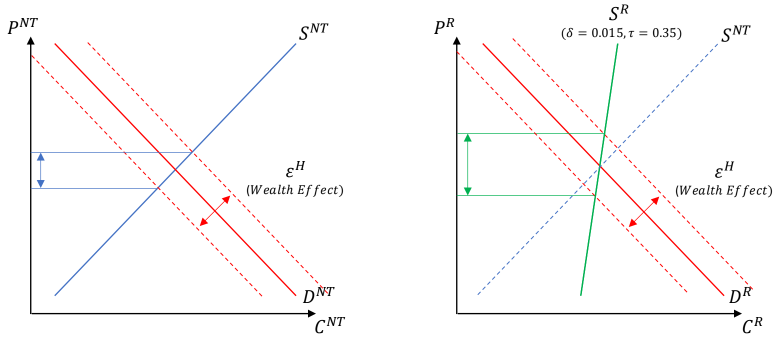

Figure 5: Wealth Effect on Nontradable (Left) and Housing Service (Right)

much more substantial for housing services because its supply curve is very inelastic. This is because housing production relies on land input, which is under fixed supply. In addition, because housing services come from housing stock, which is much larger than the per-period housing flow, even though rent (P^R) increases, increasing aggregate housing service supply is very difficult. This makes the housing services supply very inelastic, resulting in a steeper supply curve. Consequently, the rent real exchange rate responds strongly to the wealth effect.

Uniqueness of Housing Services As shown above, the two unique characteristics of housing services, land as a production input and large stock compared to flow, are important and deserve more study. To investigate the effect of such unique features further, I do comparative statics analysis on two parameters, land input share (τ) and residential structure (housing) depreciation rate (δ^S.)

Column (4) shows the results when τ changes from 0.35 to 0.01, meaning the land is not used for housing. Compared to column (2), column (4) shows a more significant cross-sectional variation of a rent real exchange rate. ($0.198 \rightarrow 0.249$) This is connected to the conventional Balassa-Samuelson hypothesis mechanism. In the model, when the home country's tradable sector productivity increases, the marginal product of production factors used in the tradable sector increases, which pushes up the other sectors' final goods prices, too, by increasing the cost of production. However, because land is not used in the tradable sector, land price is exempt from such a mechanism. Consequently, with substantially high τ, housing rent is less exposed to such a mechanism, which dampens the cross-sectional variations of rent real exchange rates.

The other interesting point in column (4) is the increased time-series variation of rent real exchange rates. ($0.038 \rightarrow 0.066$) This is because τ not only affects the slope of the supply curve as before but also affects the size of supply curve shifts caused by nontradable and construction sector productivity shocks.

Figure 6 delineates why. In the model, a positive nontradable sector productivity shock directly generates an immediate supply increase and shifts the supply curve

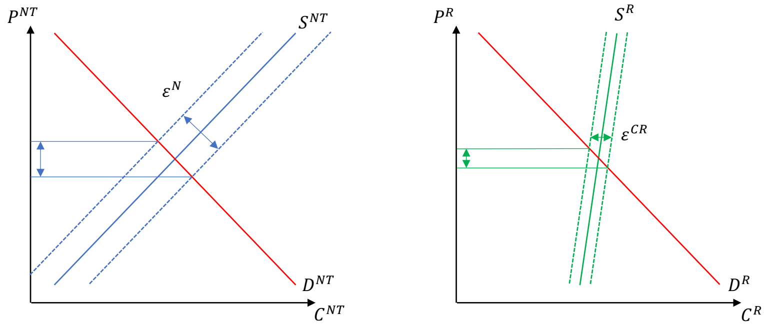

Figure 6: Productivity Shocks on the Nontradable (Left) and Housing Services (Right)

substantially. However, a positive construction sector productivity shock cannot generate a comparable shift even though the shock size is the same. The intuition is straightforward. Even though the production of construction goods (e.g., cement) increases via productivity increases, land on which to build houses is limited.[24] Consequently, the housing quantity itself cannot increase much.

This intuition is also applied to the second characteristic, a stock larger than a flow. The data shows average new house flow per year over existing houses in the eurozone area is around 0.01. This implies that even though the new housing flow doubles via advancement in housing construction technology, housing services increase only 2%. Because my model replicates such an empirical pattern well, even though the positive construction sector productivity shock hits the economy, the housing service supply does not increase more, which consequently makes the rent and rent real exchange rate fluctuate small. Column (5) shows the result when δ^S is changed from 0.00375 to 0.99, which means a flow is a new stock as all previous stock depreciates away. The time-series variations in rent real exchange rates increase substantially compared to column (2). $(0.038 \rightarrow 0.061)$[25] When $\tau = 0.01$ and $\delta^S = 0.99$, both cross-sectional and time-series variations of rent real exchange rates increase significantly, as in column (6), because both explained effects are combined. This implies that housing characteristics dampens the variations of the real exchange rate.

Role of the Cross-sectional Distribution of Sectoral Productivities A remaining puzzling observation is that even though τ is set as 0.01 and δ^S is set as 0.99, which makes housing effectively the same as other nontradable, there are still more substantial time-series and cross-section variations in q^R. It turns out that the larger time-series variation of the rent real exchange rate comes from the larger standard

[24]Urbanization and stringent land-use regulations in cities effectively limit land supply in almost every city.

[25]Cross-sectional variation of the rent real exchange rate in column (2) is not significantly different from that in column (5), which means the interval generated by their 10th quantile and 90th quantile of simulations significantly overlaps.

deviations of residuals from the construction sector productivity data used to calibrate the shock process. As in Table 7, the standard deviation of the residual from the construction sector productivity shock process is 2.564, while that of the nontradable sector is 0.963. For the cross-sectional variations, it turns out that the distribution of sectoral productivity across countries matters. Column (7) shows the simulation results when the relative construction sector productivities across countries are set as the same, which means $\frac{A_j^{CR}}{A_{EU}^{CR}} = 1$ for all j. It shows substantially dampened cross-sectional variations. (0.198 $\rightarrow$ 0.144) The remaining differences go away if I also set $\frac{A_j^{NT}}{A_{EU}^{NT}} = 1$ for all j. In summary, the distribution of sectoral productivity across countries serves a very unique role. It turns out that such distribution is closely related to the Balassa-Samuelson effect, discussed more in the next section.

4.1.2 Housing and the Balassa-Samuelson Effect

In the previous section, I examined unconditional variations of the real exchange rates. The lesson is that while housing's unique features, such as a low depreciation rate and land as a necessary input for production, are proven to decrease the variations in rent real exchange rates, the distribution of sectoral productivities across countries matters and makes rent real exchange rates larger compared to others. Note that sectoral productivities used in the simulations are estimated from actual data, as explained in Table 7. It turns out that the distribution of productivity is closely related to the Balassa-Samuelson effect. To study this, I examine the relationship between model-generated real GDP per capita (y) and real exchange rates (q, q^T, q^{NT}, q^R). In particular, I replicate the following country average cross-sectional regressions I did in the empirical analysis section by using the model-simulated real exchange rate and relative GDP per capita:

$$\begin{aligned}
\bar{q}_j &= \alpha + \beta \bar{y}_j + u_j, \\
\bar{q}_j^T &= \alpha^T + \beta^T \bar{y}_j + u_j^T, \\
\bar{q}_j^{NT} &= \alpha^{NT} + \beta^{NT} \bar{y}_j + u_j^{NT}, \\
\bar{q}_j^R &= \alpha^R + \beta^R \bar{y}_j + u_j^R.
\end{aligned}$$

Table 9 shows the β for each sector from actual data and simulations. Columns other than column (1) contain the mean value of the β from 500 simulations, and the parentheses below contain the 10th and 90th quantiles of the 500 simulations.

Data and the Baseline Model Column (1) and column (2) show how my baseline model performs compared with the data. The model overestimates the relationships between relative GDP per capita and all sectoral real exchange rates. One thing the model matches well is the relative importance of rent real exchange rates concerning

Table 9: Simulated Balassa-Samuelson Regressions: Role of Sectoral Productivity

	(1) Data	(2) Bond Baseline	(3) Arrow-Debreu Baseline	(4) Arrow-Debreu ($\gamma^R = 0.01$)	(5) Arrow-Debreu ($\tau = 0.01$)	(6) Arrow-Debreu ($\delta^S = 0.99$)	(7) Arrow-Debreu ($\bar{A}^{CR}_j / \bar{A}^{CR}_{EU} = 1$)
Bal/Sam							
β	-0.26*	-0.54⋆	-0.16⋆	-0.06	-0.18⋆	-0.18⋆	-0.13
	(0.14)	(-0.78,-0.29)	(-0.27,-0.05)	(-0.17,0.07)	(-0.30,-0.04)	(-0.30,-0.04)	(-0.25,0.00)
β^T	-0.08	-0.20⋆	-0.04	-0.03	-0.03	-0.03	-0.04
	(0.13)	(-0.32,-0.08)	(-0.11,0.03)	(-0.09,0.04)	(-0.10,0.03)	(-0.10,0.04)	(-0.12,0.04)
β^{NT}	-0.33*	-0.64⋆	-0.13	-0.09	-0.10	-0.09	-0.13
	(0.18)	(-0.99,-0.29)	(-0.33,0.09)	(-0.28,0.12)	(-0.32,0.11)	(-0.32,0.13)	(-0.37,0.12)
β^R	-0.76***	-1.52⋆	-0.61⋆	-0.49⋆	-0.83⋆	-0.65⋆	-0.38⋆
	(0.19)	(-2.09,-0.95)	(-0.83,-0.43)	(-0.69,-0.34)	(-1.24,-0.47)	(-0.98,-0.33)	(-0.45,-0.31)

The first column shows regression results from the actual data. Parentheses below the estimates include the standard errors. * means 10% significance, ** means 5% significance, *** means 1% significance. From the second column to the last column, each column shows the means of simulation-generated regression coefficients of 500 simulations. Parentheses below show the 10th and 90th quantile of 500 simulations. ⋆ implies that 0 is not in between such quantiles.

the overall Balassa Samuelson effect. Both in the data and the baseline model simulation, β^R is the largest and contributes the most to the β.

Role of a Wealth Effect As the first step, the role of the wealth effect is examined. In column (3), a complete market is assumed, and all β for sectoral real exchange rates become smaller compared to column (2), while only the β^R remains significant. This shows the role of the wealth effect in driving relative prices. Under an incomplete market, increased tradable sector productivity increases not only the income but also the price level via increasing the country's wealth and demand (Corsetti et al. 2008). Under a complete market, such a channel is lost, and price and income levels across countries show lower correlations.

Balassa-Samuelson Hypothesis and Housing Because a complete market is assumed, column (3) contains only the conventional textbook Balassa-Samuelson hypothesis mechanism. It would be interesting to examine how housing interacts with such a mechanism. A notable observation is that only β^R significantly contributes to the aggregate Balassa-Samuelson effect in column (3). If housing is abstracted away by decreasing γ^R to 0.01, the model loses its capability to generate the significant Balassa-Samuelson effect as in column (4).

To examine why housing is important for the Balassa-Samuelson hypothesis mechanism, I simulate the model by setting τ as 0.01 in column (5). Interestingly, as τ decreases, the model generates a stronger Balassa-Samuelson effect than in column (3). Aggregate β has decreased from -0.16 to -0.18, and β^R decreases from -0.61 to -0.83. This is because land (l) as an input dampens the textbook Balassa-Samuelson hypothesis mechanism. Under the Balassa-Samuelson hypothesis, when the home country's tradable sector productivity increases, the marginal product of labor (so the wage) increases, which pushes up the other sectors' marginal costs of production (so the prices.) However, land is not used in the tradable sector so that the construction sector, which has a high land input share, is less exposed to such mechanism than other nontradable sectors that use labor only.

To examine the other important characteristics of housing in the model, I set δ^S as 0.99 and simulate the model in column (6). This reveals no meaningful differences compared to column (3), which implies no special role of δ^S for the Balassa-Samuelson effect. By contrast, δ^S is important for the Backus-Smith correlation, the time-series dynamics of real exchange rates.

Role of the Cross-sectional Distribution of Sectoral Productivities The previous section showed that housing services' unique characteristics, requiring land as a production input and having a large stock compared to flow, dampen the textbook Balassa-Samuelson hypothesis mechanism through housing rents. However, the model shows that housing rent is the most significant channel through which the Balassa-Samuelson effect pattern emerges, which seems puzzling.

It turns out that this strong role of rent is from the cross-country distribution of sectoral productivities. Note that the productivity shock processes used in model simulations are calibrated directly from the EUKLEMS 2023 database. So examining the estimated mean of processes shows how sectoral productivity distribution looks across countries in actual data. Figure 7 plots the $\bar{a}_{NT}$ and $\bar{a}_{CR}$ against the $\bar{a}_H$.

From the figure, the strong correlations, $Corr(\bar{a}_H, \bar{a}_{NT}) = 0.76$ and $Corr(\bar{a}_H, \bar{a}_{CR}) =$

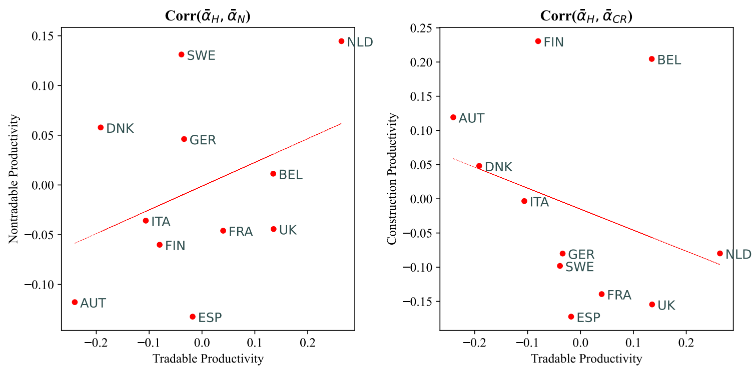

Figure 7: Cross-country Distributions of Sectoral Productivities

-0.23 appear.[26] This implies that countries with higher tradable sector productivities tend to have higher nontradable sector productivities and lower construction sector productivities.

This clearly shows why housing rent is the main channel of the Balassa-Samuelson effect in the model. Countries with higher relative tradable sector productivity tend to

[26]This figure also includes three non-eurozone countries: Sweden, Denmark, and the United Kingdom. These are estimates of the means of relative sectoral productivities from the data, and $\bar{a}_H = ln(\frac{\bar{A}_j^H}{\bar{A}_{EU}^H})$, $\bar{a}_{NT} = ln(\frac{\bar{A}_j^{NT}}{\bar{A}_{EU}^{NT}})$, $\bar{a}_{CR} = ln(\frac{\bar{A}_j^{CR}}{\bar{A}_{EU}^{CR}})$

have a higher marginal product of labor, which implies a higher wage. Then, higher wages might push up the production cost of nontradable services, pushing up the nontradable prices. However, as shown in Figure 7, these countries also tend to have higher nontradable sector productivity which lowers the marginal cost of nontradable sector production. In the end, nontradable does not show much of a price hike. On the other hand, countries with higher relative tradable sector productivities tend to have lower relative construction sector productivities. So, while the wages are high, the marginal cost of construction goods gets even higher because of low construction sector productivity.

Once it is counterfactually assumed that all countries' construction sector productivities are the same (i.e., $\bar{A}_j^{CR}/\bar{A}_{EU}^{CR} = 1$ for all j), as in column (7), the simulated β is not significant anymore, and the absolute size of β^R increases from -0.61 to -0.38. Though not added in the table, if $\bar{A}_j^{NT}/\bar{A}_{EU}^{NT} = 1$ for all j is assumed additionally, β^{NT} and β^{CR} become very similar to each other, removing the prominent role of housing rents. This means that the effect of the negative correlation between the tradable and construction sector productivity outweighs the dampening effect from τ, land.

This corresponds to recent literature on stagnant productivity in the construction sector. Goolsbee and Syverson (2023) show that construction sector productivity has been decreasing in the US and the EU 27 area as a whole. If the construction sector is the sector whose productivity does not grow in all countries while other nontradable sector productivity increases, the different growth rates of tradable sector productivity across countries should generate a strong Balassa-Samuelson effect via the rent real exchange rate. Because there are observations of the productivity of only eleven countries' in this study, more research on a more granular level is necessary to link the Balassa-Samuelson effect and construction sector productivity across regions.

4.1.3 Housing and the Backus-Smith Puzzle

Lastly, I examine the role of housing in the Backus-Smith puzzle via model simulation. Using model-simulated data, I replicate the following four-panel regressions as in the empirical analysis section:

$$
\begin{aligned}
\Delta q_{jt} &= \alpha + \beta \Delta c_{jt} + e_{jt}, \\
\Delta q_{jt}^T &= \alpha^T + \beta^T \Delta c_{jt} + e_{jt}^T, \\
\Delta q_{jt}^{NT} &= \alpha^{NT} + \beta^{NT} \Delta c_{jt} + e_{jt}^{NT}, \\
\Delta q_{jt}^R &= \alpha^R + \beta^R \Delta c_{jt} + e_{jt}^R.
\end{aligned}
$$

Table 10 presents replication results under different calibrations. First, column (2) clearly shows that the model cannot replicate the negative β under a complete market. It generates β close to σ, calibrated as 2. It is because the complete market condition implies that $ln(C_t/C_t^*) = \frac{1}{\sigma}ln(P_t^*/P_t)$ for every state and time, demonstrating the Backus-Smith puzzle found by Backus and Smith (1993). Moving to column (3), I assume an incomplete market but a very small housing expenditure share in the model by setting γ^R as 0.01. Though the model generates 0.47, which is much smaller than the case of column (1), it is still very far from its data counterpart, being statistically significantly different from 0. This again shows why the Backus-Smith puzzle is hard to resolve, even under the incomplete market assumption.

One of the main findings appears in column (4), where I set γ^R as 0.25 as in the baseline calibration, which generates a realistic rent expenditure share of 17% in the model. Now the model can replicate negative β, whose point estimate is -0.01. In addition, such a negative correlation primarily comes from the β^R, whose value is -1.18. In addition, increased γ^R decreases the value of β^T and β^{NT}, making them closer to the empirical estimates, helping the model to match other sectoral real exchange rates' behaviors as well. Once the γ^R increases to 0.45, the model generates much more negative β as in column (5), which underscores the role of the housing sector.

Table 10: Simulated Backus-Smith Puzzle Regressions: Role of Housing

	(1) **Data**	(2) **Arrow-Deberu** (γ^R = 0.25)	(3) **Bond** (γ^R = 0.01)	(4) **Bond** (γ^R = 0.25)	(5) **Bond** (γ^R = 0.45)
Coefficients					
β	-0.14**	1.99⋆	0.47⋆	-0.01	-0.60⋆
	(0.07)	(1.98,2.02)	(0.11,0.83)	(-0.36,0.41)	(-1.09,-0.11)
β^T	0.02	1.09⋆	0.25	0.12	-0.01
	(0.05)	(1.08,1.10)	(0.05,0.45)	(-0.08,0.34)	(-0.28,0.25)
β^{NT}	-0.15**	3.36⋆	0.83⋆	0.42	0.03
	(0.06)	(3.33,3.40)	(0.23,1.44)	(-0.18,1.11)	(-0.77,0.84)
β^R	-0.53**	0.82⋆	$-0.70^\star$	$-1.18^\star$	$-1.78^\star$
	(0.23)	(0.81,0.83)	(-1.07,-0.33)	(-1.54,-0.76)	(-2.26,-1.29)

The first column shows regression results from the actual data. Parentheses below the estimates include the standard errors. * means 10% significance, ** means 5% significance, *** means 1% significance. From the second column to the last column, each column shows the means of simulation-generated regression coefficients of 500 simulations. Parentheses below show the 10th and 90th quantile of 500 simulations. ⋆ implies that 0 is not in between such quantiles.

To understand why a realistically calibrated housing sector helps the model generate a negative Backus-Smith correlation, it is necessary to examine how model-generated real exchange rates and relative consumption respond to each sectoral productivity shock, which is the sole source of the business cycles in the model. Figure 8 shows the impulse response functions (IRF) of the real exchange rate (q) and the relative consumption (c) to a one standard deviation relative sectoral productivity shock.

One notable observation is that the tradable sector shock decreases q (appreciates the real exchange rate) and increases c (increases the relative consumption) while the nontradable sector shock and construction sector shock increase q (depreciate the real exchange rate) and increase c. Given that c moves in the same direction for all types of shocks, a sign of the model-generated Backus-Smith correlation, $Corr(\Delta c, \Delta q)$, will

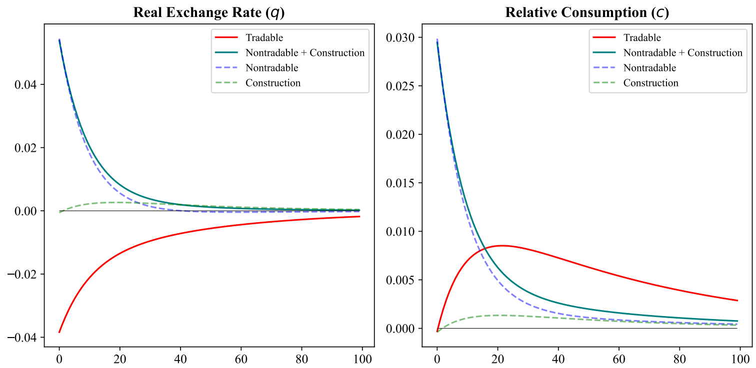

Figure 8: Model q and c Responses to Sectoral Productivity Shocks

depend on the relative size of the effect of tradable sector shock on q compared with those of the nontradable and construction sector shock.

As the mechanism behind the effect of the nontradable and construction sector shock are similar for the Backus-Smith correlation, it is easier to understand once these two forces are aggregated into one force represented by the teal-colored line in Figure 8, resulting in two forces counteracting each other in the model. One is the tradable sector shock generating $Corr(\Delta c, \Delta q) < 0$, and the other is the sum of the nontradable and construction sector shocks generating $Corr(\Delta c, \Delta q) > 0$.

Given these characteristics of the shocks, showing how the IRFs of q and c to such shocks change under different housing weights is the most straightforward way to check the role of housing in the Backus-Smith correlation. Figure 9 shows how these model responses change when γ^R changes from 0.01 to 0.45 in the model.

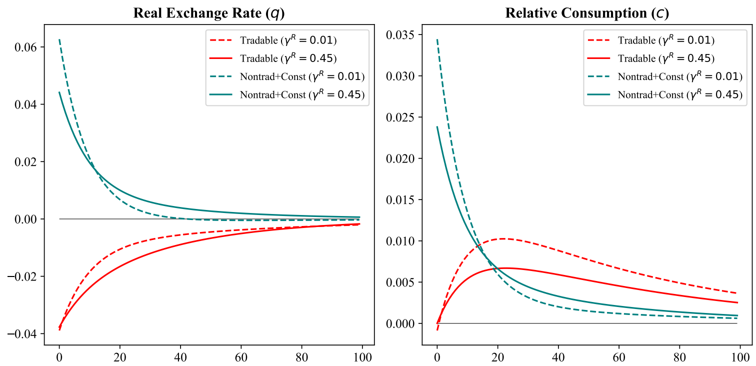

Figure 9: Role of Housing in IRFs of q and c

Comparing the dotted line ($\gamma^R = 0.01$) with the solid line ($\gamma^R = 0.45$), it is observed that the IRFs of q to all shocks are dragged down to lower values with higher γ^R. In particular, while the effects of the nontradable and construction sector shocks get smaller than before, the effect of the tradable sector shock gets more persistent. This implies that under higher γ^R the tradable sector shock effect gets amplified and the nontradable and construction sector shock effect gets smaller, causing the model to generate a more negative aggregate correlation between Δq and Δc.

Note that Δq is an expenditure-weighted average of $\Delta q^T, \Delta q^{NT}$, and Δq^R in the model since we used the first-order approximation method for the model solution. This means that as γ^R gets larger, Δq will be affected more by Δq^R but less by Δq^T and Δq^{NT}. So understanding the response of each sectoral real exchange rate's IRFs is essential. Figure 10 shows the IRFs of sectoral real exchange rates to both the tradable sector shock (ϵ_h) and the sum of the nontradable and construction sector shocks ($\epsilon_n + \epsilon_{cr}$) under both a complete and incomplete market.

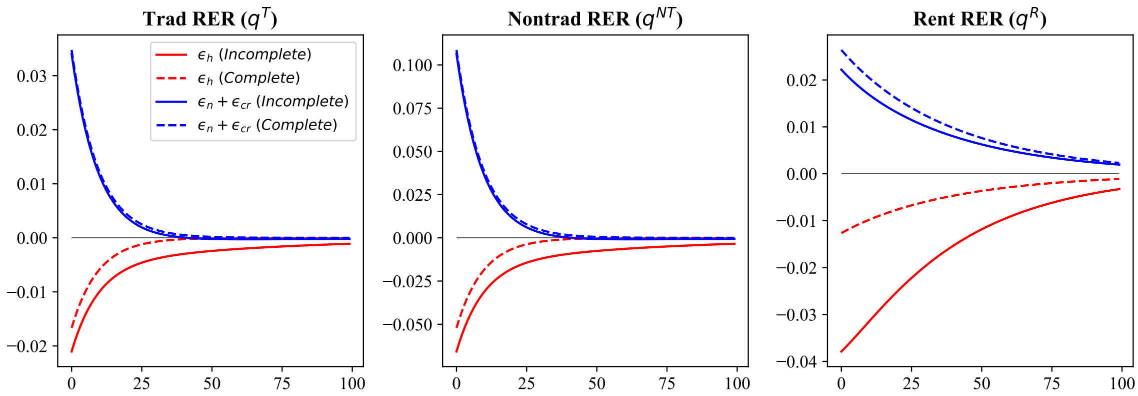

Figure 10: Impulse Response Functions of Sectoral Real Exchange Rates

Housing and Tradable Sector Productivity Shock First, we focus on the effect of the tradable sector shock (ϵ_h). All sectoral real exchange rates appreciate responding to the positive tradable sector productivity shock. Two forces generate such appreciations. The first mechanism is the well-known textbook Balassa-Samuelson hypothesis. When the tradable sector gets a positive productivity shock, it increases the marginal product of labor and pushes up wages. This increased wage in turn increases the marginal production cost of nontradables and housing construction via the labor market. Note that q^T also appreciates due to the distribution margin. The other mechanism is the wealth effect generated under an incomplete market. If the home country receives a positive tradable sector productivity shock, the home country becomes wealthier than the foreign country and consumes more than the foreign country, increasing the home country's demand for all goods and services more than that of the foreign country, resulting in the home appreciation.[27]

While the Balassa-Samuleson hypothesis channel works under any risk-sharing assumption, the wealth effect works only under an incomplete market, implying that the difference between IRFs under a complete market and an incomplete market can be interpreted as the wealth effect. In Figure 10, it becomes clear that the q^R responds

[27]To make such a wealth effect work, a model calibration should be within a specific parameter region featuring high substitutability between tradable goods. Our calibration is within such a region. For more detail, refer to Corsetti et al. (2008).

much more to the wealth effect than to other sectoral real exchange rates. The difference between the red dotted line and the red solid line is asymmetrically more significant for q^R compared with q^T and q^{NT}. This arises from the fact that housing services supply is more inelastic than tradable or other nontradable, as shown in Figure 5. Furthermore, the q^R's impulse response function is the most persistent because of slower adjustment in the aggregate supply of housing services.

On the other hand, comparing the dotted IRF of q^{NT} and q^R shows that the textbook Balassa-Samuelson hypothesis mechanism works much less for q^R. q^{NT} appreciates almost 5%, while q^R appreciates only by 1%. This is because of land. While land is important for housing production, it is not used in the tradable sector. Consequently, even though the tradable sector receives a positive productivity shock, it does not increase the price of land. Naturally, this leads to a smaller increase in the production cost of housing compared with that of the nontradable.

In summary, the unique characteristics of housing services generate asymmetry between the responses of q^R and those of q^T or q^{NT} to the change in the tradable sector productivity shock. This implies that when γ^R increases, the response of q arises more from q^R, which generates a stronger response to the tradable sector shock and more negative Backus-Smith correlation forces.

Housing and Nontradable/Construction Sector Productivity Shock While it is understandable that q^T shows little response to nontradable and construction sector shocks since its productivity is not affected, strikingly q^R is not depreciating as much as q^{NT}. This recalls Figure 6. The large land input share (τ) and low depreciation rate of housing stock (δ^S) decrease the effect of the construction sector productivity shock on the supply of housing services. Consequently, the supply increase is limited, and housing services prices do not decrease much. If $\tau = 0$ and $\delta^S = 1$, the responses of q^R would be exactly the same as those of q^{NT}.

In summary, the unique characteristics of housing services generate the asymmetry in which the response of q^R to the productivity shock on its sector is much smaller than that of q^{NT}. This implies that when γ^R increases, q's response is more driven by that of q^R, which makes q respond less to the sum of productivity shocks in the nontradable and construction sectors. In aggregate, this causes the model to generate less positive Backus-Smith correlation forces.

Housing, Inelastic Supply, and the Backus-Smith Puzzle In general equilibrium, two roles of housing explained above work simultaneously when the γ^R gets larger in the model. Consequently, the housing sector causes the model to generate a more negative Backus-Smith correlation in aggregate.

Table 11 shows how simulation results change when housing-related parameters change. Column (2) is the baseline case where γ^R is 0.25, generating results similar to the data. When τ changes to 0.01, in column (3), the model generates a stronger negative Backus-Smith correlation. This is because the tradable sector shock gets amplified via a stronger Balassa-Samuelson hypothesis mechanism with a smaller land-input share. In addition, the very low δ^S limits its dampening effect on the nontradable and construction sector. And this leads to a stronger negative Backus-Smith correlation.

Column (4) is where I set δ^S as 0.99 so that housing services have an elastic supply like other nontradables. Then, the model generates positive β and even positive β^R. In this case, housing services no longer show dampened responses to the construction sector productivity shocks since their prices decrease dramatically once they receive the positive productivity shocks. In addition, q^R's significant response to the wealth effect via the tradable sector shock will disappear. Consequently, the model loses its capacity to generate a negative β estimate. In column (5), once I assume both low τ and high δ^S, the model gets further from the negative β. In column (3), because of low δ^S, τ could not play its role in dampening the response of q^R to the construction productivity shock. Given the high δ^S, high τ was doing its job by dampening the construction sector shocks, but once τ is also set low, its role disappears, generating a stronger positive β than in column (4). These simulations show that the inelastic housing supply and its interaction with the wealth effect and nontradable/construction sector productivity shocks are the key mechanisms for generating the negative Backus-Smith correlation in the model.

Table 11: Simulated Backus-Smith Puzzle Regressions: Role of Housing

	(1) **Data**	(2) **Bond** **Baseline**	(3) **Bond** ($\tau = 0.01$)	(4) **Bond** ($\delta^S = 0.99$)	(5) **Bond** (τ=0.01, δ^S=0.99)
Backus/Smith					
β	-0.14** (0.07)	-0.06 (-0.45,0,32)	-0.35 (-0.75,0.03)	0.61⋆ (0.20,1.07)	1.17⋆ (0.80,1.55)
β^T	0.02 (0.05)	0.09 (-0.12,0.29)	-0.09 (-0.31,0.11)	0.29⋆ (0.09,0.51)	0.30⋆ (0.11,0.46)
β^{NT}	-0.15** (0.06)	0.33 (-0.31,0.97)	-0.23 (-0.88,0.38)	0.95⋆ (0.33,1.61)	0.94⋆ (0.39,1.44)
β^R	-0.53** (0.23)	-1.24⋆ (-1.61,-0.86)	-1.47⋆ (-2.00,-0.98)	0.84⋆ (0.15,1.55)	3.47⋆ (2.68,4.28)

The first column shows regression results from the actual data. Parentheses below the estimates include the standard deviations. * means 10% significance, ** means 5% significance, *** means 1% significance. From the second column to the last column, each column shows the means of simulation-generated regression coefficients of 500 simulations. Parentheses below show the 10th percentile and 90th percentile of 500 simulations. ⋆ implies that 0 is not in between the 10th percentile and 90th percentile of the simulated estimates.

It is important to note that this is a more general result than it seems. In this paper, I use housing services as a representative example because of their economic significance and unique characteristics. However, any goods and services can work similarly in the model as long as they account for a large expenditure share (share of the overall price level), their supply is very inelastic, and they require unique production factors that are immune to productivity gains.

Discussion of Corsetti et al. (2008) and Benigno and Thoenissen (2008) It is worthwhile to discuss the relationship between our findings and the lessons of Corsetti et al. (2008) and Benigno and Thoenissen (2008). First, Corsetti et al. (2008) resolve the Backus-Smith puzzle under an incomplete market by making either the tradable goods very non-substitutable and pushing up the terms of trade or by making the productivity shock itself very persistent under high substitutability of the tradable goods, causing the wealth effect itself to be stronger. Our work is the same as that of Corsetti et al. (2008), in that it uses the wealth effect via an incomplete market. However, our model differs in how it amplifies the relative price responses to such a wealth effect. Rather than affecting the terms of trade or increasing the persistence of the shock itself as in Corsetti et al. (2008), I make the response of the real exchange rates stronger by making the aggregate supply more inelastic, and such inelasticity comes from incorporating inelastically supplied housing services in the aggregate consumption with a significant expenditure share.

Second, Benigno and Thoenissen (2008) resolve the Backus-Smith puzzle by increasing the nontradable prices via the Balassa-Samuelson hypothesis mechanism, using the tradable sector productivity shock. Our model does not depend on such a mechanism. Because land is used only in housing production, the Balassa-Samuelson hypothesis mechanism is weak in our model. Being different from them, our approach focuses more on dampening the effect of construction sector productivity shocks by using the unique characteristics of housing. In a standard model, positive productivity shock in the nontradable sector (including the construction sector) generates the positive Backus-Smith correlation because it increases supply of goods, increasing the consumption and lowering the prices relative to the foreign country. However, in our model with housing, because housing is challenging to supply due to its large land input share and low depreciation rate, any positive productivity gain in the construction sector cannot increase the supply. Consequently, the model generates less positive Backus-Smith correlation compared to the standard model.

Our model also differs from the sector-specific capital model. In the sector-specific capital model, though those sector-specific capitals are hard to adjust, they are not the only input for producing those sectors' output. These sectors also use labor, which makes the supply elastic. However, housing services come only from housing stock, not additional labor. This causes the supply to be dramatically inelastic, which helps the model improve on the Backus-Smith puzzle.

5 Conclusion

This paper has examined the role of the housing sector in international business cycles, with a specific focus on its role in real exchange rate dynamics. Using disaggregated relative price level data from eurozone countries, I show that relative rent is the most volatile component of the aggregate real exchange rate. Moreover, the rent real exchange rate contributes to over half of the Balassa-Samuelson effect and the negative Backus-Smith correlation within eurozone countries.

Building on these empirical findings, I construct a two-country, three-sector model with a realistically calibrated housing sector. The simulation of the model using sectoral productivity shocks directly calibrated from the EUKLEMS database yields several insights into the roles of real rent exchange rates and the housing sector. Including a realistically calibrated housing sector enables the model to generate greater cross-sectional and time-series variations.

Furthermore, housing characteristics, such as the role of land and the large stock compared with the relatively small flow, have been identified as factors that mitigate the textbook Balassa-Samuelson hypothesis mechanism. The model demonstrates that the strong Balassa-Samuelson effect via q^R stems from the negative correlation between relative productivities of the tradable sector and the construction sector.

Lastly, the model incorporating the housing sector yields improved predictions for the Backus-Smith correlation. The inelastic housing supply intensifies the model's response to wealth effects (demand shocks) and mitigates its response to nontradable and construction sector productivity shocks. These mechanisms have shifted the responses of aggregate real exchange rates for all shocks to a negative direction, which helps the model generate a negative Backus-Smith correlation as in the data.

These implications not only shed light on the role of the housing sector in real exchange rate dynamics within eurozone countries but also offer broader insights into the functioning of international business cycle models. Although housing rent has been used as a representative example due to its economic significance, the underlying principles revealed in this study apply to any goods and services characterized by limited productivity growth, reliance on unique production inputs not used in other sectors, or inelastic supply. Concerning addressing the Balassa-Samuelson effect, further exploration of stagnant construction sector productivity observed in the recent literature will be crucial. In the context of the Backus-Smith puzzle, it would be valuable to investigate heterogeneity among countries' expenditure weights on inelastically supplied goods and services.

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